

Week in Review # 1

MATH 141
1.1 through 1.4

Drost-Spring 2010

1. Find the distance between $(-4, 3)$ and $(1, 5)$.

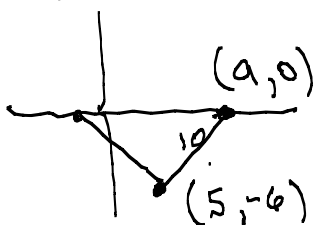
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(1 + 4)^2 + (5 - 3)^2}$$

$$d = \sqrt{25 + 4} = \sqrt{29}$$

2. Find the coordinates of the point(s) that are 10 units away from the point $(5, -6)$ and are on the x -axis.

draw a picture



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$10 = \sqrt{(a - 5)^2 + (0 + 6)^2}$$

square both sides

$$100 = (a - 5)^2 + 36$$

$$64 = (a - 5)^2$$

$$\pm 8 = a - 5$$

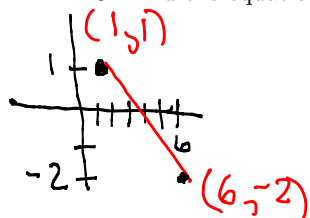
$$\pm 8 + 5 = a$$

$$a = 8 + 5 = 13$$

$$\text{or } a = -8 + 5 = -3$$

$$a = 13, -3$$

3. Find the equation of the circle whose center is $(6, -2)$ which passes through the point $(1, 1)$.



Equation of a circle

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - 6)^2 + (y + 2)^2 = 34$$

$$(h, k) = (6, -2)$$

$$r = \sqrt{34}$$

$$r = \sqrt{(6 - 1)^2 + (-2 - 1)^2}$$

$$r = \sqrt{25 + 9} = \sqrt{34}$$

4. Find the value of a such that the distance from $(-3, 3)$ to $(a - 1, 6)$ is 5.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\pm 4 - 2 = a$$

$$5 = \sqrt{(a - 1 + 3)^2 + (6 - 3)^2}$$

$$a = 4 - 2 = 2$$

or

$$25 = (a + 2)^2 + 9$$

$$a = -4 - 2 = -6$$

$$16 = (a + 2)^2$$

$$\pm 4 = a + 2$$

$$a = 2, -6$$

5. Graph: $y = \sqrt{1-x^2}$

$$y^2 = 1 - x^2$$

$$x^2 + y^2 = 1$$

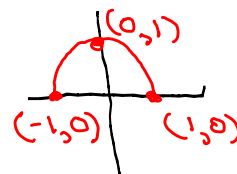
$$(x-h)^2 + (y-k)^2 = r^2$$

$$(h,k) = 0$$

$$r = 1$$

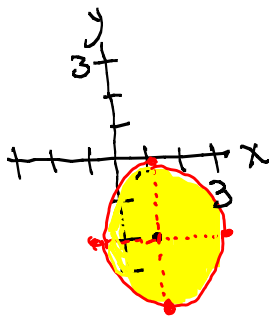
original eq $y = \sqrt{1-x^2}$
 $y = +\sqrt{\quad}$

So the graph is the top half of the circle \Rightarrow



6. Graph: $(x-1)^2 + (y+2)^2 \leq 4$

circle with center at $(1, -2)$ with radius 2



test a point to decide whether to shade inside \odot or outside \odot

test pt: $(1, -2)$

$$(1-1)^2 + (-2+2)^2 \leq 4$$

$$0 \leq 4$$

true

7. Find the slope between $(-3, a+1)$ and $(a-1, -1)$

$$m = \frac{\Delta y}{\Delta x} = \frac{a+1 - (-1)}{-3 - (a-1)} = \frac{a+2}{-3-a+1} = \frac{a+2}{-a-2}$$

$$m = \frac{a+2}{-1(a+2)} = \boxed{-1}$$

8. Given $l_1 : 3x + 2y = 4$

a) Write the equation of l_2 in slope-intercept form, which is and passes through the point

b) Write the equation of l_3 in general form, which is and passes through the point $(-2, 3)$.

$$l_1: 3x + 2y = 4$$

$$2y = -3x + 4$$

$$y = -\frac{3}{2}x + 2$$

$$m_1 = -\frac{3}{2}$$

a) parallel to l_1

$$m_2 = -\frac{3}{2}$$

$$(5, -1)$$

$$y - y_1 = m(x - x_1)$$

$$y - (-1) = -\frac{3}{2}(x - 5)$$

$$y + 1 = -\frac{3}{2}x + \frac{15}{2}$$

$$\boxed{y = -\frac{3}{2}x + \frac{13}{2}}$$

b) perpendicular to l_1 ,

$$m_3 = \frac{2}{3}$$

$$(-2, 3)$$

$$y - 3 = \frac{2}{3}(x + 2)$$

mult by 3

$$3y - 9 = 2(x + 2)$$

$$3y - 9 = 2x + 4$$

$$\boxed{0 = 2x - 3y + 13}$$

9. Find the value of a such that AB is perpendicular to CD . $A = (-2, 5)$, $B = (a + 2, 2)$, $C = (-1, -2)$, $D = (3, 6)$

$$m_{AB} = \frac{5-2}{-2-(a+2)}$$

$$m_{CD} = \frac{6+2}{3+1}$$

$$\frac{6}{-a-4} = -1$$

$$m_{AB} = \frac{3}{-a-4}$$

$$m_{CD} = \frac{8}{4} = 2$$

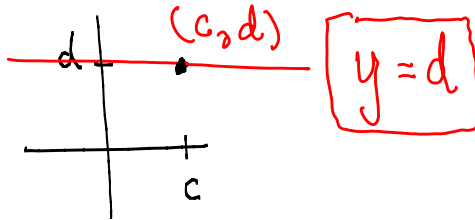
$$6 = -1(-a-4)$$

$$6 = a+4$$

$$AB \perp CD \text{ if } m_{AB} \cdot m_{CD} = -1 \Rightarrow \frac{3}{-a-4} \cdot 2 = -1$$

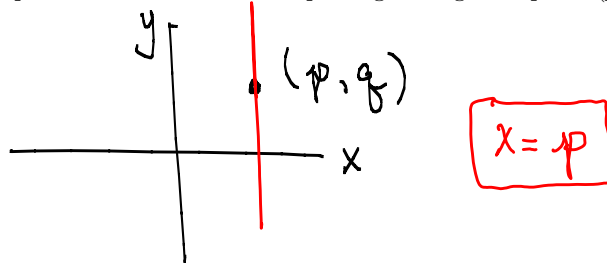
$$\boxed{2=a}$$

10. Write the equation of the horizontal line passing through the point (c, d) .



$$\frac{6}{-a-4} = -1$$

11. Write the equation of the vertical line passing through the point (p, q) .



12. Using data from professor P, 26% of the students taking exam 1 will earn a B on the test.

- find an equation that expresses the relationship between s , the number of students taking exam one, and q , the number of students who earn a B.
- if there are 150 students in the class, how many will earn a B?
- if 91 students scored a B, how many students took the exam?

$s = \# \text{ students}$

a) $\boxed{q = .26s}$

$q = \# \text{ who made a B}$

b) if $s = 150$, find q

$$q = 0.26(150) = \boxed{39 \text{ students}}$$

c) if $q = 91$, find s

$$91 = 0.26s$$

$$\frac{91}{0.26} = s$$

$$s = \boxed{350 \text{ students}}$$

13. A company has fixed costs of \$3500 and production costs of \$5 per item produced. The product sells for \$12/unit.

- What is the cost function?
- What is the revenue function?
- What is the profit function?
- What is the profit or loss when 1500 items are produced and sold?

$$f = 3500$$

$$v = 5$$

$$p = 12$$

a) $C = f + vx$

$$C = 3500 + 5x$$

b) $R = x \cdot p$

$$R = 12x$$

c) $P = R - C$

$$P = 12x - (3500 + 5x)$$

$$P = 7x - 3500$$

d) if $x = 1500$

$$P = 7(1500) - 3500$$

$$P = 7000$$

$$\boxed{\$7000}$$

14. Company XYZ sells computers for \$1200 each. To produce fifty computers the total costs are \$44,350. The company spends \$845 to build each computer. Write the equation for the profit function.

$$p = 1200$$

$$x = 50$$

$$C = 44,350$$

$$v = 845$$

$$R = x \cdot p$$

$$R = 1200x$$

$$C = f + v \cdot x$$

$$44350 = f + 845(50)$$

$$44350 = f + 42,250$$

$$2100 = f$$

$$C = 2100 + 845x$$

$$P = R - C$$

$$P = 1200x - (2100 + 845x)$$

$$P = 1200x - 2100 - 845x$$

$$\boxed{P = 355x - 2100}$$

15. At a price of \$48, the quantity demanded is 650 units. At a price of \$60, the quantity demanded is 350. Given that it is linear,

- find the demand equation.
- At what price is there no demand?
- What quantity is demanded if free?

price-demand
 (x, p)
 $(650, 48)$
 $(350, 60)$

$$m = \frac{60 - 48}{350 - 650}$$

$$m = \frac{12}{-300} = -\frac{4}{100}$$

$$y - y_1 = m(x - x_1)$$

$$p - p_1 = m(x - x_1)$$

$$p - 60 = -\frac{4}{100}(x - 350)$$

$$p - 60 = -\frac{4}{100}x + 14$$

$$\boxed{p = -\frac{4}{100}x + 74}$$

no demand
 $x = 0$

$$p = -\frac{4}{100}(0) + 74$$

$$p = 74$$

$$\boxed{\$74}$$

find x if $p = 0$

$$0 = -\frac{4}{100}x + 74$$

$$\frac{4}{100}x = 74$$

$$\boxed{x = 1850}$$

16. Producers will make 1000 refrigerators available when the unit price is \$210. At a unit price of \$260, 3000 refrigerators will be marketed.

- Find the supply equation.
- How many will be marketed when the price is \$310?
- What is the lowest price at which a refrigerator will be marketed?

supply-demand

$$m = \frac{260 - 210}{3000 - 1000} = \frac{50}{2000} = \frac{1}{40} = 0.025$$

(x, p)

$(1000, 210)$

$(3000, 260)$

$$p - p_1 = m(x - x_1)$$

$$p - 210 = \frac{1}{40}(x - 1000)$$

$$p - 210 = \frac{1}{40}x - 25$$

$$\boxed{p = \frac{1}{40}x + 185}$$

b) if $p = 310$

$$310 = \frac{1}{40}x + 185$$

$$125 = \frac{1}{40}x \Rightarrow \boxed{x = 5,000}$$

c) if $x = 1$, $p = \frac{1}{40}(1) + 185$

$$p = 185.025 \quad \boxed{\$185.03}$$

17. Find the intersection of:

$$y = 3x + 20 \text{ and}$$

$$-y = 2x - 10$$

$$\begin{cases} y = 3x + 20 \\ -y = 2x - 10 \end{cases}$$

add equals $0 = 5x + 10$

to equals $-10 = 5x$

$$-2 = x$$

$$y = 3x + 20$$

$$y = 3(-2) + 20$$

$$y = -6 + 20$$

$$y = 14$$

$$\boxed{(-2, 14)}$$

18. Find the break-even point if

$$C(x) = 6x + 1200 \text{ and}$$

$$R(x) = 10x$$

$$C = 6x + 1200$$

$$R = 10x$$

Break-even $R = C$

Break-even point $(300, 3000)$

$$10x = 6x + 1200$$

$$4x = 1200$$

$$x = 300$$

break even

Quantity

$$R = 10x$$

$$R = 10(300)$$

$$R = 3000$$

break-even
price

19. Rent-a-Van leases vehicles at \$35/day plus 25cent/mile. Rent-an-SUV leases vehicles at \$28/day and 50cent/mile.

a) Write a daily cost function for each company.

b) If the customer plans to drive 50 miles, which is less expensive?

c) How much cheaper is it?

$m = \# \text{ miles}$

a) $R_{aV} = 35 + 0.25m$

$$R_{aSuv} = 28 + 0.50m$$

b) if $m = 50$

$$R_{aV} = 35 + 0.25(50) = \$47.50$$

$$R_{aSuv} = 28 + 0.50(50) = \$53.00$$

Rent-a-Van is cheaper, by \$5.50.

20. Given a demand equation $x + 3p - 600 = 0$ where X is the quantity demanded monthly, and p is the price in dollars. The supply equation is $x - 2p - 200 = 0$ where x is the quantity the supplier will provide when the price is p dollars.

a) Find the equilibrium price.

b) Find the equilibrium quantity.

demand

$$x + 3p - 600 = 0$$

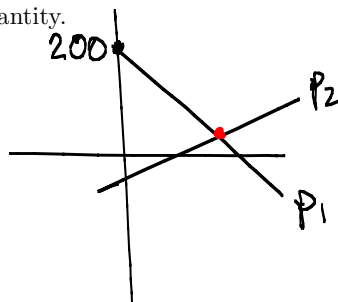
$$3p = -x + 600$$

$$p_1 = -\frac{1}{3}x + 200$$

Supply $x - 2p - 200 = 0$

$$x - 200 = 2p$$

$$\frac{1}{2}x - 100 = p_2$$



$$\begin{cases} p = -\frac{1}{3}x + 200 \\ p = \frac{1}{2}x - 100 \end{cases}$$

$$p = \frac{1}{2}x - 100$$

$$-\frac{1}{3}x + 200 = \frac{1}{2}x - 100$$

mult by 6

$$-2x + 1200 = 3x - 600$$

$$1800 = 5x$$

$$360 = x$$

$$p = \frac{1}{2}x - 100$$

$$p = \frac{1}{2}(360) - 100 = 80$$

$$\boxed{\begin{matrix} a) \$80 \\ b) 360 \end{matrix}}$$

100

100

100

100

100

100

100

100

100

100

100

100

100

100

