Week in Review # 10

MATH 141 Drost-Spring 2010

8.2, 8.3, 8.4

1. The chef at the local Mexican restuarant recorded the weekly consumption of salsa over a 20 week period.

Qts Salsa	100	120	140	160	180	200	Sum
Weeks	1	2	5	5	4	3	20

a) Find the average weekly consumption of salsa at this restuarant.

b) Let X denote the number of quarts of salsa consumed in a week at the restuarant. Find the probability distribution of the random variable X and compute $E(X)$, the expected value of X.

a) $$\text{average} = \frac{[100(1)+120(2)+140(5)+160(5)+180(4)+200(3)]}{20}$$

average = 158 quarts of Salsa/wk

b)

X	100	120	140	160	180	200
$P(X=x)$	$\frac{1}{20}$	$\frac{2}{20}$	$\frac{5}{20}$	$\frac{5}{20}$	$\frac{4}{20}$	$\frac{3}{20}$

$$E(x) = 100\left(\frac{1}{20}\right) + 120\left(\frac{1}{10}\right) + 140\left(\frac{1}{4}\right) + 160\left(\frac{1}{4}\right) + 180\left(\frac{1}{5}\right) + 200\left(\frac{3}{20}\right)$$

$$E(x) = 5 + 12 + 35 + 40 + 36 + 30$$

$$E(x) = 158 \text{ quarts}$$

2. If a sample of 4 batteries is selected from a lot of 20 batteries, of which 2 are defective, what is the expected number of defective batteries?

X = number of defective batteries

$$P(X=0) = \frac{C(18,4)\cdot C(2,0)}{C(20,4)} = \frac{12}{19}$$

$$P(X=1) = \frac{C(18,3)\cdot C(2,1)}{C(20,4)} = \frac{32}{95}$$

$$P(X=2) = \frac{C(18,2)\cdot C(2,2)}{C(20,4)} = \frac{3}{95}$$

x	0	1	2
$P(X=x)$	$\frac{12}{19}$	$\frac{32}{95}$	$\frac{3}{95}$

$$E(X) = 0\cdot\frac{12}{19} + 1\cdot\frac{32}{95} + 2\cdot\frac{3}{95} = \boxed{0.4}$$

3. Solar Designs built a spec house at a cost of \$275,000. They estimate they can sell the house for \$400,000, \$425,000, or \$450,000 with probabilities of 0.30, 0.45, and 0.25, respectively. What is the expected profit for Solar Designs on this particular model?

x =	400,000	425,000	450,000
$P(X=x)$ =	0.3	0.45	0.25

$$E(X) = 400{,}000(0.3) + 425{,}000(0.45) + 450{,}000(0.25)$$

$$E(X) = \$423{,}750$$

Expected Sales price \$423,750
cost to build − 275,000
Expected profit \$148,750

4. Compare two car dealerships with the following weekly volume of sales, with corresponding probabilities:

All Star Autos

X

# Sold/Wk	4	5	6	7	8	9	10
Probability	.04	.12	.25	.30	.15	.10	.04

Best Car Deals

Y

# Sold/Wk	4	5	6	7	8
Probability	.08	.13	.26	.35	.18

The average profit/car at *All Star Autos* is \$520, and the average profit/car at *Best Car Deals* is \$640.

a) Find the average number sold each week at each dealership.

$$E(X) = 4(.04) + 5(.12) + 6(.25) + 7(.30) + 8(.15) + 9(.10) + 10(.04)$$

$E(X) = 6.86$ Cars/wk sold

$$E(Y) = 4(.08) + 5(.13) + 6(.26) + 7(.35) + 8(.18)$$

$E(Y) = 6.42$ cars/wk sold

b) Which dealership has the highest expected weekly profit?

Profit ASA $= 6.86(520) = \$3{,}567.20$

Profit BCD $= 6.42(640) = \$4{,}108.80$

$\boxed{\text{Best Car Deals}}$

5. Suppose the probability that it will rain tomorrow is 0.3.

a) What are the odds that it will rain tomorrow?

b) What are the odds that it will **not** rain tomorrow?

$P(r) = \frac{3}{10}$

$P(r^c) = \frac{7}{10}$

a) odds it will rain =

$$\frac{P(E)}{1-P(E)} = \frac{.3}{1-.3} = \frac{.3}{.7} = \frac{3}{7}$$

b) odds it will not rain

$$\frac{1-P(E)}{P(E)} = \frac{.7}{.3} = \frac{7}{3}$$

6. The odds against an event E occurring are 4 to 5. What is the probability of E not occurring?

$$\text{odds } E^c = \frac{4}{5} = \frac{1-P(E)}{P(E)}$$

$$4P(E) = 5(1-P(E))$$

$$4P(E) = 5 - 5P(E)$$

$$9P(E) = 5$$

$$P(E) = \frac{5}{9}$$

$P(E) = \frac{5}{9}$

$P(E^c) = \frac{4}{9}$

7. The following scores were earned on a recent multiple choice history exam given to 16 students.

~~65~~	~~75~~	~~50~~	~~95~~
~~80~~	~~95~~	~~100~~	~~35~~
~~40~~	~~75~~	~~60~~	~~85~~
~~60~~	~~95~~	~~55~~	~~70~~

100 95 85 80 75 70 65 60
95 75 60
95

55 50 40 35

Find the mean, mode, and median score on the ecxam.

X =	100	95	85	80	75	70	65	60	55	50	40	35
freq	1	3	1	1	2	1	1	2	1	1		

$$\text{a) mean} = \frac{\text{sum of scores}}{\#\text{ of scores}} = \frac{1135}{16}$$

$$= 70.9375\% \approx 70.94\%$$

b) mode = 95%

$$\text{c) median} = \frac{75+70}{2} = 72.5\%$$

Variance and Standard Deviation

8. Find the variance of the random variable X in problem number 4 above for *All Star Autos*.

$$E(X) = 6.86 = \mu$$

$$Var(X) = p_1(x_1-\mu)^2 + p_2(x_2-\mu)^2 + \ldots$$

$$Var(X) = .04(4-6.86)^2 + .12(5-6.86)^2 + .25(6-6.86)^2 + .30(7-6.86)^2 + .15(8-6.86)^2 + .10(9-6.86)^2 + .04(10-6.86)^2$$

$$Var(X) = 1.9804$$

9. Find the standard deviation of the random variable X for *All Star Autos*.

$$\sigma = \sqrt{Var(X)}$$

$$\sigma = \sqrt{1.9804} \approx 1.4073$$

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10. In a class of 100 students, define X as the random variable whose value represents the students grade in the finite math class, $A = 4$, $B = 3$, etc. Compute the mean, and standard deviation.

$X =$	4	3	2	1	0	Sum
freq. of occurrance	13	30	42	10	5	= 100
$P(X=x)$	.13	.30	.42	.10	.05	

$$E(X) = 4(.13) + 3(.3) + 2(.42) + 1(.10) + 0(.05)$$

$$E(X) = 2.36 = \mu$$

$$\bar{x} = \frac{4(13) + 3(30) + 2(42) + 1(10) + 0(5)}{100}$$

$$\bar{x} = 2.36 = \mu$$

$$\sigma = \sqrt{Var(X)}$$

$$Var(X) = p_1(x_1 - \mu)^2 + p_2(x_2 - \mu)^2 + \dots$$

$$Var(X) = .13(4 - 2.36)^2 + .3(3 - 2.36)^2 + .42(2 - 2.36)^2 + .1(1 - 2.36)^2 + .05(0 - 2.36)^2$$

$$Var(X) = 0.9904$$

$$\sigma = \sqrt{0.9904}$$

$$\sigma = 0.9952 \quad \text{standard deviation}$$

11. A probability distribution has a mean of 18 and a standard deviation of 1.3. Use Chebychev's inequality to find a bound on the probability that an outcome of the experiment lies between 14 and 22.

$\mu - K\sigma = 14$

$18 - 1.3K = 14$

$-1.3K = -4$

$K = \frac{4}{1.3}$

$K = \frac{40}{13}$

$\bar{x} = 18 = E(X) = \mu$

$\sigma = 1.3$

$P(\mu - K\sigma \le x \le \mu + K\sigma) \ge 1 - \frac{1}{K^2}$

$P(14 \le x \le 22) \ge 1 - \frac{1}{\left(\frac{40}{13}\right)^2}$

$\ge 1 - \frac{13^2}{40^2}$

≥ 0.894375

$\mu + K\sigma = 22$

$18 + 1.3K = 22$

$1.3K = 4$

$K = \frac{40}{13}$

12. Find the variance of the probability distribution:

$X =$	1	2	3	4	5
$P(X) =$	0.1	0.3	0.3	0.2	0.1

$Var(X) = p_1(x_1 - \mu)^2 + p_2(x_2 - \mu)^2 + \ldots + p_5(x_5 - \mu)^2$

$\mu = E(x) = x_1 \cdot p_1 + x_2 \cdot p_2 + \ldots + x_5 p_5$

$E(X) = 1(.1) + 2(.3) + 3(.3) + 4(.2) + 5(.1)$

$E(X) = .1 + .6 + .9 + .8 + .5 = 2.9$

$Var(X) = .1(1 - 2.9)^2 + .3(2 - 2.9)^2 + .3(3 - 2.9)^2$

$+ .2(4 - 2.9)^2 + .1(5 - 2.9)^2$

$Var(X) = 1.29$

13. The minimum age requirement for a regular driver's licence varies in each state.

Minimum Age	14	15	16	17	18
freq. of occurrance	2	7	39	1	1

Source: http://www.2pass.co.wk/ages2.htm

a) Describe a random variable X that is associated with this data.

X = denotes the minimum age of a regular drivers licence

b) Find the probability distribution for the random variable X.

$X =$	14	15	16	17	18
$P(X=x)$	$\frac{2}{50}$	$\frac{7}{50}$	$\frac{39}{50}$	$\frac{1}{50}$	$\frac{1}{50}$

c) Compute the mean, variance, and standard deviation of X.

$$\text{Mean} = \frac{14(2)+15(7)+16(39)+17(1)+18(1)}{50}$$

Mean = 15.84

$$\text{Variance} = \frac{2}{50}(14-15.84)^2 + \frac{7}{50}(15-15.84)^2 + \frac{39}{50}(16-15.84)^2 + \frac{1}{50}(17-15.84)^2 + \frac{1}{50}(18-15.84)^2$$

Variance = 0.3744

Standard deviation

$$\sigma = \sqrt{0.3744}$$

$$\sigma = 0.6119$$

14. The number of married men (in thousands) between the ages of 20 and 44 in the United States in 1998 is given in the following table:

	22	27	32	37	42	Sum
Age	20-24	25-29	30-34	35-39	40-44	
Men	1332	4219	6345	7598	7633	27,127

Find the mean and the standard deviation of the given data. Assume that all scores lying within a group interval take the middle value of that group. *Source: FINITE MATH, by Tan, p. 449, #24*

$$\bar{x} = \frac{\left[22(1332) + 27(4219) + 32(6345) + 37(7598) + 42(7633)\right]}{27{,}127}$$

$$\text{mean} = \bar{x} = 34.94558927\ldots$$

$$\bar{x} \approx 34.9456 = E(X)$$

Probability Distribution

$x =$	22	27	32	37	42
$P(X=x) =$	.0491	.15553	.2339	.28009	.28138

$$Var(X) = p_1(x_1-\mu)^2 + p_2(x_2-\mu)^2 + \cdots$$

$$Var(X) = .0491(22-34.9456)^2 + .15553(27-34.9456)^2 + .2339(32-34.9456)^2 + .28009(37-34.9456)^2 + .28138(42-34.9456)^2$$

$$Var(X) = 35.26194064\ldots$$

Standard deviation

$$\sigma = \sqrt{35.2619} \approx 5.9382$$

Binomial Distribution

15. A fair eight-sided die is rolled 2 times. If a 3 or a 5 lands uppermost in a trial, then the throw is considered a *success*. Otherwise, the throw is considered a *failure*.

X = binomial random variable representing # of successes

a) Find the probability of obtaining exactly 0, 1, or 2 successes.

fixed number of trials

$n = 2$

$P(3 \text{ or } 5) = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$ $p = \frac{1}{4}$ $q = \frac{3}{4}$

$P(X=x) = C(n,x)\, p^x q^{n-x}$

$P(X=0) = C(2,0) \frac{1}{4}^0 \left(\frac{3}{4}\right)^2 = \frac{9}{16}$

$P(X=1) = C(2,1) \frac{1}{4}^1 \left(\frac{3}{4}\right)^1 = \frac{3}{8}$

$P(X=2) = C(2,2) \frac{1}{4}^2 \left(\frac{3}{4}\right)^0 = \frac{1}{16}$

b) Construct a binomial distribution and draw a histogram.

X	0	1	2
$P(X=x)$	$9/16$	$3/8$	$1/16$

c) What is the probability of obtaining 0 or 1 success in the experiment?

$P(X=0 \text{ or } X=1) = \frac{9}{16} + \frac{3}{8} = \boxed{\frac{15}{16}}$

d) What is the probability of obtaining at least one success in the experiment?

$P(\text{at least } 1) = 1 - P(X=0) = 1 - \frac{9}{16} = \boxed{\frac{7}{16}}$

e) Compute the mean, variance, and standard deviation of X.

$\mu = E(X) = n \cdot p = 2\left(\frac{1}{4}\right) = \boxed{\frac{1}{2}}$

$Var(X) = n \cdot p \cdot q = 2\left(\frac{1}{4}\right)\left(\frac{3}{4}\right) = \boxed{\frac{3}{8}}$

$\sigma = \sqrt{npq} = \sqrt{\frac{3}{8}} = \boxed{\frac{\sqrt{6}}{4}}$