Week in Review # 13

MATH 141 Drost-Spring 2010

5.1, 5.2, 5.3, 9.1

5.1 Compound Interest

$I = Prt$

$A = P + I = P + Prt$ or $A = P(1 + rt)$

Compound Interest Formula

$A = P(1 + i)^n, \quad u = \frac{r}{m}, \quad n = m \cdot t$

$A = P(1 + \frac{r}{m}^{mt}$

r=nomial rate/year

m= # conversions/year

t=# years

P= principal

Continuous Compound Interest Formula

$A = Pe^{rt}$

Effective Rate of Interest

effective rate = annual % yield

$r_{eff} = (1 + \frac{r}{m})^m - 1$

Present Value

$P = A(1 + i)^{-n} = A(a + \frac{r}{m})^{-n}$

5.2 Annuities

ordinary annuity : payments made at the end of each payment period

annuity due: payments made at the beginning of each payment period

simple annuity: payment period coincides with the interest conversion period

complex annuity: payment period differs from the interest conversion period

In 5.2, all annuities will have

a) fixed time intervals

b) periodic payments equal in size

c) payments made at end of payment period

d) payment period coincides with interest conversion periods

The future value S of an annuity

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$

n payments

$\$R$ each payment

paid at the end of each investment period into an account with interest rate of i per period $(i = \dfrac{r}{m})$

Present Value of an Annuity

$$P = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

5.3 Amortization of Loans

$$R = \frac{Pi}{1-(1+i)^{-n}}$$

payment of R dollars

on a loan of $\$P$

over n periods, $(n = m \cdot t)$

with interest i per period $(i = \frac{r}{m})$

using the **TVM Solver**

$N = \#$ payments

$I\% =$ rate

$PV = \$$ borrowed $=$ *enter as* $-\$$

$PMT = \boxed{\text{repayment}} => \$\$$

$FV = m$

$P/Y = m$

Amortization Schedules

End of Period	Interest	Repay-ment	Payment Toward Principal	Outstanding Principal
0				$P_0 = \$$ borrowed
1	$i_1 = P_o rt$	\$\$	\$\$-interest	$P_1 = \$ - (\$\$ - i)$
2	$i_2 = P_1 rt$	\$\$	\$\$ $-i_2$	$P_2 = P_1 - (\$\$ - i_2$

9.1 Markov Chains

stochastic: must be a square matrix, with each entry positive, and the sum of the entries in each column is 1.

QUESTIONS of the Week

1. Find the amount of money at the end of 8 years on a \$500 deposit in an account paying simple interest at a rate of 3.25% per year. How much interest is earned?

a) $A = P(1 + rt)$

$A = 500(1 + .0325(8))$

$\boxed{A = \$630}$

b) $I = A - P = 630 - 500$

$\boxed{I = \$130}$

2. A savings account paying simple interest grew from an initial deposit of \$3250 to a sum of \$3347.50 in 8 months. Find the interest rate.

$A = P(1 + rt)$

$3347.50 = 3250\left(1 + r \cdot \frac{8}{12}\right)$

$\frac{3347.50}{3250} = 1 + \frac{2}{3}r$

$.03 = \frac{2}{3}r$

$\frac{3}{2}(.03) = r$

$r = .045 = \boxed{4.5\%}$

3. How much money will be in an account after 3 years on a \$1,000 deposit that earns 4.25% per year compounded continuously?

$A = Pe^{rt}$

$t = 3$

$P = 1000$

$r = .0425$

$A = 1000e^{(.0425)(3)} = \boxed{\$1,135.98}$

4. If Mike invests \$5000 into an account paying interest at a rate of $5\frac{3}{4}\%$ per year compounded quarterly, how much money will he have at the end of 5 years (assuming no additional deposits or withdrawals)?

$$A = P\left(1+\frac{r}{n}\right)^{mt}$$

$$A = 5000\left(1+\frac{.0575}{4}\right)^{4(5)}$$

$$\boxed{A = \$6{,}651.82}$$

or TVM Solver

N = 4(5)

I% = 5.75

PV = −5000

PMT = 0

FV = $\boxed{}$

P/Y = 4

C/Y = 4

5. In 18 years, Jane and Bill need \$60,000 for their daughter's college expenses. If they find an account paying interest at a rate of 5.5% per year compounded weekly, how much should they invest now, in order to meet their goal of \$60,000?

TVM

N = 12(18)

I% = 5.5

PV = $\boxed{}$ = $\boxed{\$22{,}306.27}$

PMT = 0

FV = 60,000 P/Y = C/Y = 12

6. What interest rate, compounded bi-weekly, will triple \$4000 in only 9 years?

r = ?

n = 52(2)

PV = x

FX = 3x

t = 9

N = 52(2)·9

I% = $\boxed{}$ = $\boxed{12.21\%}$

PV = −4000

PMT = 0

FV = 12000

P/Y = C/Y = 52(2)

7. A major credit card company has a finance charge of 1.75% per month on the outstanding balance. Jim charged \$1450 on his bill, and made no payments for 9 months. What is the balance on the account after 9 months?

$$\frac{1.75\%}{\cancel{\text{month}}}\left(\frac{12\ \cancel{\text{months}}}{1\ \text{year}}\right) = \frac{21\%}{\text{yr}}$$

TVM

$N = 12\left(\frac{9}{12}\right)$

$I\% = 21$

$PV = -1450$

$PMT = 0$

$FV = \boxed{}$

$P/Y = C/Y = 12$

$$\boxed{FV = \$1469.14}$$

8. Which account would be the best option for an investment?

Option A: 8% per year, compounded weekly

Option B: 7.85% per year, compounded daily

Option C: 7.9% per year, compounded continuously

$$r_{eff} = \left(1 + \frac{r}{m}\right)^m - 1$$

A. $r_{eff} = \left(1 + \frac{.08}{52}\right)^{52} - 1 = .08322\ldots \approx \boxed{8.32\%}$

eff(%, m)

B. $r_{eff} = \left(1 + \frac{.0785}{365}\right)^{365} - 1 = .08165\ldots \approx 8.17\%$

eff(%, m)

C. $r_{eff} = \underbrace{e^r - 1}_{\text{continuously compounded}} = .0822\ldots \approx 8.22\%$

9. If Matthew deposits $125 at the end of each month into a savings account earning interest at a rate of 4.25% per year compounded monthly, how much will he have at the end of 10 years (assuming that he makes no withdrawals during that period)? How much interest will he earn?

N = 10(12)

I% = 4.25

PV = 0

PMT = −125

FV = $\boxed{}$ = $\boxed{\$18,650.91}$

P/Y = C/Y = 12

10. I am looking to purchase a new car and have $2500 for a down payment. The car I want has a cash price of $38.750. If the best financing option you find charges interest at a rate of 4.75% per year compounded monthly, what would the monthly payments be in order to pay off the loan in 60 months. b) In 48 months?

N = 60

I% = 4.75

PV = −36250

PMT = $\boxed{}$

FV = 0

P/Y = C/Y = 12

price $38,750.

down − 2,500.

loan $36,250

4.75% m=12 60 months = 5 yr

a. PMTS = $\boxed{\$679.94}$

b, in 48 months

(change N = , re-calculate)

PMTS = $\boxed{\$830.71}$

11. Mary Jane bought a car after making a down payment of \$5475, and secured financing for the balance of the purchase price at a rate of 4.5% per year compounded weekly. Under the terms of the finance agreement, she is required to make payments of \$465.00 each month for 48 months.

a. What was the purchase price of the car?

b. How much interest does she pay with this financing?

price = 5475 + loan

TVM Solver

N = 48

I% = 4.5

PV = $\boxed{}$ = loan = \$21,853.52

PMT = −465

FV = 0

P/Y = C/Y = 52

price = 5475 + loan amt

price = 5475 + 21,853.52

price = $\boxed{\$27,328.52}$

b) Total paid = (465)(48) = \$22,320

= \$. #

$$\begin{array}{r} \$22,320.00 \\ -\ 21,853.52 \\ \hline \boxed{\$466.48} \end{array}$$

12. A sum of $15,000 is to be repaid over a 4 year period through equal installments made at the end of each year. If an interest rate of 5.25% is charged on the unpaid balance and interest calculations are made at the end of the year, determine the size of each installment so that the load is amortized at the end of 4 years. Show the amortization schedule.

TVM Solver

$N = 4$
$I\% = 5.25$
$PV = 15{,}000$
$PMT = \boxed{} = 4254.77$
$FV = 0$
$P/Y = C/Y = 1$

Amortization Schedule

End of Period	Pmts Remain	Payments	To Interest	To Principal	Outstanding Principal	Equity
0	4	0	–	–	15,000	0
1	3	4254.77	787.50	3467.27	11,532.73	3467.27
2	2	4254.77	605.47	3649.30	7,883.43	7,116.57
3	1	4254.77	413.88	3840.89	4,042.54	10,957.46
4	0	4254.77	212.23	4042.54	0	15,000

13. Paul buys a new \$3200 HDTV with 3D, by paying \$500 down and financing the rest. The terms of the finance agreement state that the unpaid balance will be charged interest a rate of 18% per year, compounded monthly, and the money is to be repaid over a 2-year period through equal installments made at the end of each month.

a. What will Paul's monthly payments be? $\boxed{\$134.80}$

$N = 2(12) = 24$

$I\% = 18$

$PV = -2700$

$PMT = \boxed{} = \$134.80$

$FV = 0, \; P/Y = C/Y = 12$

b. How much of the first payment goes towards paying down the loan?

$\frac{18\%}{\cancel{\text{year}}} \cdot \frac{1\ \cancel{\text{year}}}{12\ \text{month}} = \frac{1.5\%}{\text{month}}$

3200 price
− 500 down
2700 loan

$2700(.015)(1) =$

$I = Prt = \$40.50 = \text{interest}$

134.80 pmt
− 40.50 interest
$\boxed{\$94.30 \text{ to principal}}$

c. How much equity will Paul have in his HDTV after 1 year?

after 1 year ⇒ 12 pmts remain

TVM Solver

$N = 12$

$I\% = 18$

$PV = \boxed{} = \$1470.33$ = amt still due

$PMT = -134.80$

$FV = 0$

$P/Y = C/Y = 12$

3200
− 1470.33
$\boxed{\$1729.67 \text{ equity}}$

14. Twelve years ago John made a down payment on a house of 10% of the purchase price and secured a bank loan of \$135,000 to finance the remaining amount. The mortgage was for a term of 25 years, with an interest rate of 6.75% per year compounded monthly on the unpaid balance to be amortized through equal monthly payments.

a. What is the outstanding principal on John's house now?

x = price of house in \$

$.90x = 135{,}000$

$x = 150{,}000$

House bought 12 yrs ago

Calculate pmt
TVM
$N = 12(25)$
$I = 6.75$
$PV = -135{,}000$
\$932.73 = $PMT = \boxed{}$
$FV = 0$
$P/Y = C/Y = 12$

after 12 yrs,
13 yrs of pmts left.
TVM Solver
$N = 12(13)$
$I\% = 6.75$
$PV = \boxed{} \Rightarrow \boxed{\$96{,}698.01}$
$PMT = -932.73$
$FV = 0, \quad P/Y = C/Y = 12$

b. How much equity does John have in the house now?

Equity = total value − amt still owed

Equity = 150,000 − 96,698.01

$\boxed{\text{Equity} = \$53{,}301.99}$

c. How much total interest will John pay over the life of the loan?

Total paid = \$932.73(25)(12) = \$279,819

loan = 135,000

$\boxed{\text{interest } \$144{,}819}$

15. Mark has started to think about his retirement. He has decided that his goal is to receive quarterly payments of \$6,000 for 15 yrs when he retires at age 65. To achieve this goal, Mark is going to make quarterly payments into his retirement account for forty years. He just turned 25. His retirement account earns interest at a rate of 6%/yr compounded quarterly.

What payments should he make to meet his goal?

TVM Solver

$N = 40(4)$

$I\% = 6$ $\quad$ $FV = 360{,}000$

$PV = 0$ $\quad$ $P/Y = C/Y = 4$

$\boxed{\$549.42} = PMT = \boxed{}$

16. Your credit card has a balance of \$2,000 and the interest rate is 1.95% compounded monthly. If you only make the minimum monthly payment of \$40 per month, how many years will it take to pay off the balance?

b) How much interest did you pay? Assume there are no new charges to the card.

$N = \boxed{} = mt = 191.0118084 = mt$

$I\% = 1.95(12)$ $\quad$ $191.0118084 = 12t$

$PV = -2000$ $\quad$ $\boxed{15.9 \text{ yrs} = t}$

$PMT = 40$

$FV = 0$ $\quad$ $\$40^{00}(15.9)(12) - 2000$

$P/Y = C/Y = 12$

$\$5632 - 2000 = \boxed{\$3632 \text{ interest}}$

17. Rework the previous problem with monthly payments of \$50.

$N = \boxed{} = mt$

$I\% = 1.95(12)$ $\quad$ $\boxed{t = 6.5 \text{ years}}$

$PV = -2000$

$PMT = 50$ $\quad$ $\$50(6.5)(12) - 2000$

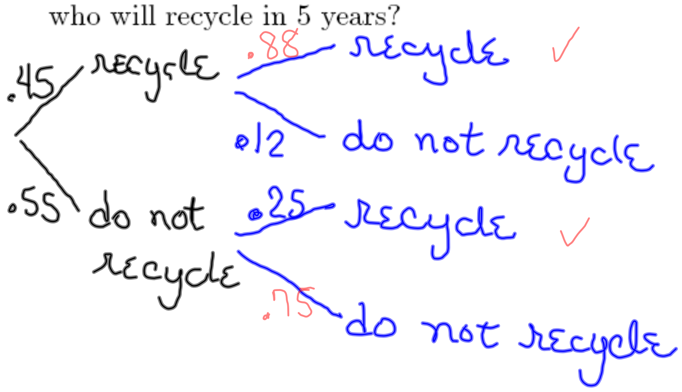
$FV = 0$

$P/Y = C/Y = 12$

$\$3900$
-2000

$\boxed{\$1900 \text{ interest}}$

18. Among homeowners in College Station, 45% recycle, and in 5 years, 12% of those who recycle will have dropped out of the program, and 25% of those who didn't recycle, will in 5 years. What is the expected distribution of the homeowners who will recycle in 5 years?



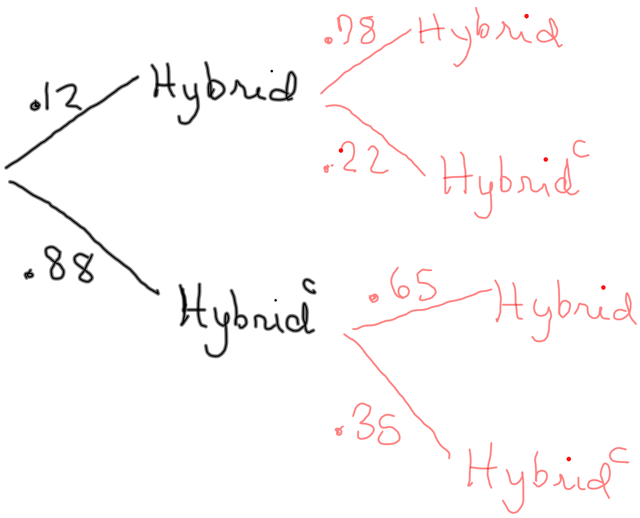
$p = .45(.88) + .55(.25) = 0.5965$

59.65% will recycle in 5yrs.

$$T = \begin{bmatrix} .88 & .25 \\ .12 & .75 \end{bmatrix} \qquad X_0 = \begin{bmatrix} .45 \\ .55 \end{bmatrix}$$

$$T \cdot X_0 = X_1 = \begin{bmatrix} .5965 \\ .4035 \end{bmatrix} \begin{matrix} \text{will recycle} \\ \text{NOT recycle} \end{matrix}$$

19. A poll of automobile owners in 1990 showed that 12% of the owners drove a hybrid vehicle. A trend has been shown that after a decade, 78% of the owners of hybrid vehicles replaced their vehicle with another hybrid car, and 35% of the owners who did not drive hybrids replaced them ten years later with another gasoline powered car. Determine the expected distribution in 2010.



$$T = \begin{bmatrix} .78 & .65 \\ .22 & .35 \end{bmatrix}$$

$$X_0 = \begin{bmatrix} .12 \\ .88 \end{bmatrix}$$

$$X_1 = TX_0 = \begin{bmatrix} .6656 \\ .3344 \end{bmatrix}$$

$$X_2 = T \cdot X_1 = \begin{bmatrix} .7365 \\ .2635 \end{bmatrix}$$ hybrid

73.65%

$\boxed{\text{Note}}$

$X_2 = T^2 \cdot X_0$