

Week in Review # 14

MATH 141

Drost-Spring 2010

9.1, 9.2, Final Exam Review

9.1 Markov Chains

stochastic matrix

all entries are nonnegative (≥ 0)

all entries in each column add up to 1

transition matrix T

a square matrix $n \times n$

Transition matrix (with three states)

$$T = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix}$$

$$\text{initial state} = X_o = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}$$

$$X_1 = T \cdot X_o$$

$$X_2 \cdot T \cdot X_1 \text{ or } X_2 = T^2 \cdot X_o$$

9.2 Regular Markov Chains

definition : if the sequence T, T^2, T^3, \dots approaches a steady-state matrix in which the columns of the limiting matrix are all equal, and all entries are positive.

QUESTIONS of the Week

1. The transition matrix for a Markov process is

given by $T = \begin{bmatrix} \frac{1}{5} & \frac{3}{10} \\ \frac{4}{5} & \frac{7}{10} \end{bmatrix}$ and the initial-state

distributions vector is given by $X_o = \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \end{bmatrix}$

Find $X_1 = TX_o$ and interpret with the aid of a tree diagram.

2. Find $X_2 = TX_1$ and show it is equivalent to T^2X_o .

3. Records show the percentage of students who change their major each year at State U. are as follows in the transition matrix below. The current freshman class has 38% Business majors, 22% Humanities majors, 25% Engineering and Science majors, and 15% General Studies majors.

	<i>Bus</i>	<i>Hum</i>	<i>Eng/Sci</i>	<i>Gen.Stud.</i>
$T =$	<i>B</i>	<i>H</i>	<i>E/S</i>	<i>GS</i>
	.75	.05	.15	.25
	.05	.80	.10	.40
	.15	.10	.60	.10
	.05	.05	.15	.25

Assuming these trends continue, find the percentage of these students in each major their senior year.

4. A survey of a thousand students showed 30% used AT&T, 20% used T-Mobile, 10% Sprint, and 40% used Verizon. The following transition matrix represents transitional probabilities from one plan to another every two years, when their service contract expired.

		AT&T	<i>TMobile</i>	<i>Sprint</i>	<i>Verizon</i>
$T =$	AT&T	.75	.05	.10	.05
	<i>TMobile</i>	.10	.80	.05	.15
	<i>Sprint</i>	.15	.05	.85	.10
	<i>Verizon</i>	0	.10	0	.70

Assuming these trends continue, what is the expected distribution in four years?

5. Determine which of the following matrices are regular.

a. $\begin{bmatrix} .6 & .8 \\ .4 & .2 \end{bmatrix}$

b. $\begin{bmatrix} 1 & .2 \\ 0 & .8 \end{bmatrix}$

c. $\begin{bmatrix} .4 & .2 & .5 \\ .3 & .4 & .1 \\ .3 & .5 & .4 \end{bmatrix}$

d. $\begin{bmatrix} .3 & 1 \\ .7 & 0 \end{bmatrix}$

6. Find the steady-state distribution vector for the regular Markov chain whose transition matrix vector is $T = \begin{bmatrix} .45 & .25 \\ .55 & .75 \end{bmatrix}$

7. Hypothetical data, (made up by me), indicates that over the next five years, 35% of those who currently own a gasoline powered vehicle will switch to a hybrid, and 15% of those who currently drive a hybrid will switch to a gasoline powered vehicle. Currently in my home town, 32% of the vehicles are hybrids.

a. If this trend continues, what percentage of cars will be hybrids in ten years?

b. In the long run, what percentage of cars will be hybrid?

8. Currently 40% of the viewers watch Fox Sports, 35% watch ESPN, and 25% watch CBS Sports. Studies show the following transitional matrix for these sports fans:

		Fox Sports	ESPN	CBS Sports
$T =$	Fox Sports	.6	.3	0
	ESPN	.4	.4	.6
	CBS Sports	0	.3	.4

a. If these trends continue, what percentage will be watching *ESPN* in three years?

b. In the long run, what share of the audience will each network command?

Final Exam Review Questions, Part A

1. Solve for x , y , and z in the following matrix equation.

$$\begin{bmatrix} 2 & -1 \\ 3 & -1 \\ x & 4 \end{bmatrix} - 2 \begin{bmatrix} y-3 & 2 \\ 5 & 5-z \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} 4 & -5 \\ -7 & -5 \\ 3 & -8 \end{bmatrix}$$

2. Suppose the random variable $X = \#$ of cards drawn without replacement from a standard deck, if a card is drawn until a diamond appears. What are the values of X and what is its classification (finite discrete, infinite discrete, or continuous)?
3. Find the standard deviation (rounded to the nearest dollar) of the following list of salaries (in thousands of dollars) at a small company. Assume that the salaries cover every employee at the company.

<i>Salary</i>	32	38	45	48	55
<i>Frequency</i>	12	20	14	3	1

4. In a poll of seven-hundred students, 450 used Facebook, 375 were on Twitter, and 125 were on both. What is the probability that a student from this group, selected randomly, is on Twitter but not Facebook? All the students polled were on one or both of these social networks.
5. A system of linear equations has the following matrix.

$$\begin{array}{cccc|c} x & y & z & & N \\ \hline 1 & 1 & 2 & & 3 \\ 0 & 1 & 1 & & -2 \\ 0 & 0 & 1 & & 4 \end{array}$$

- a. How many solutions does the system have?
- b. Solve the system.
6. The set $A = \{0, 1, 2, 3, 4, 5\}$. Label each of the following statements as true, or false.
- $\phi \subset A$
 - $\{2, 4\} \in A$
 - $0 \subset A$
 - The set A has 16 proper subsets.

7. $A = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 3 & 2 \\ 0 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 5 & -2 \\ 3 & -1 \\ 0 & 4 \end{bmatrix}$, and

$$C = \begin{bmatrix} -3 & 4 & 2 \\ 4 & -3 & 1 \\ 1 & -1 & 0 \end{bmatrix},$$

- a. Find the matrix $D = (A - C)B$
- b. What is D_{22} ?

8. Find the equation of the line through the point $(2, -3)$ that is perpendicular to the y -axis.
9. The frequency distribution of the random variable X is shown below.

X	-2	0	2	4	6
<i>freq</i>	15	12	5	10	8

- a. Write the probability distribution.
- b. Find $P(X > 0)$.
10. If you roll a fair six-sided die 50 times, what is the probability that you will roll at most ten sixes?

11. You start a savings account for a trip to Greece, by depositing \$250 in an account paying 3.75% interest compounded weekly. What payments should you add to the account in order to have saved \$5800 in three years?

12. For the demand equation $4x + 3p - 52 = 0$ and the supply equation $5x - 2p - 19 = 0$, x represents the quantity of a product in units of 100, and p is the unit price in dollars. How many units of the product must be produced in order to reach market equilibrium?

13. $S = \{a, b, c, d, e, f\}$, $A = \{a, b, c, d\}$,

$$B = \{a, b, d\}, C = \{d, e, f\}$$

- a. Describe by roster method the set

$$D = (A \cap B \cap C)^c.$$

- b. Describe by roster method the set

$$E = A^c \cup B^c$$

14. What is the **exact** distance between the points $(-2, 4)$ and $(3, 7)$?
15. In the 1999-2000 academic year, the schools in the Big 12 Conference saw the following athletic performances. 5 schools had winning records in Soccer, 7 schools had winning records in Women's Basketball, 8 schools had winning records in Volleyball, 2 schools had winning records in all 3 sports, 2 schools had winning records only in Women's Basketball, 2 schools had winning records only in Women's Basketball and Volleyball, and every school had a winning record in at least one of these sports. How many schools had winning records in Soccer and Volleyball, but not Women's Basketball?
16. You invest \$2400 for 5 years in an account that earns 3.25% annual interest compounded quarterly. Find the effective rate of this account.
17. Find the x value, rounded to two decimal places, that satisfies $P(X > x) = 0.8907$ if $\mu = 69.4$ and $\sigma = 13.23$.

18. A box contains 2 red, 4 green, and 5 yellow marbles. In how many ways can a sample of 3 marbles be chosen without replacement if there are no restrictions?

19. The Lemon Wedge (opening this summer in Bryan) has a choice of 12 entrees and 5 desserts. If you plan to order an a entree and a dessert, how many different two-course meals are possible?

20. A business machine purchased for \$8,000 is sold as scrap for \$1,500 after 5 years of use. Find the value of the machine, after two years of use.

21. The probability that Jim will live another year is 0.9935. He can purchase a \$35,000 one-year term life insurance policy for \$400. What is the insurance company's expected gain if Jim purchases this policy?

22. A student is picking their fall schedule from the following offerings: 3 Math courses, 4 English courses, 2 Political Science courses, 6 History courses, and 5 Kinesiology courses. If the student wants to take one of each course, how many options for a fall schedule will the student have?

23. Suppose $P(E) = 0.4$, $P(F) = 0.6$ and $P(E \cup F) = 0.8$. Calculate:

- a. $P(E \cap F)$
- b. $P(F^c)$
- c. $P(E^c \cup F^c)$
- d. $P(E^c \cap F)$

24. A company manufactures 40% of its hard drives in Austin, 25% in Beaumont, and 35% in Corpus Christi. Quality control estimates that 1% of all hard drives produced in Austin, 3% of all hard drives in Beaumont, and 2% of all hard drives in Corpus Christi are defective. If a hard drive chosen at random is found to be defective, what is the probability that it was manufactured in Austin?

25. Find the equation of the least-squares line of the following data.

x	2	5	6	8	11
y	1	3	4	10	12

26. Graph (using reverse shading) the region described by the inequalities:

$$\begin{aligned} x + 2y &\leq 10 \\ 5x + 3y &\leq 30 \\ x &\geq 0, y \geq 0 \end{aligned}$$

27. How many distinguishable arrangements of the letters in the words "Hullabaloo" are possible?

28. The following data indicates the number of sofas, x , which would be sold at the given price p in dollars.

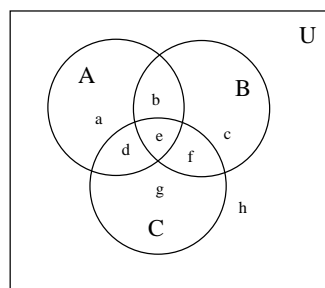
x	100	200	300	400	500
p	150	130	90	60	10

If the demand is linear and the supply equation is given by $p = 0.1x + 22$, what is the equilibrium price?

29. Determine the solution of the linear system given below:

$$\begin{cases} 2x - 4y = 4 \\ 3x + 2y = 6 \end{cases}$$

30. From the Venn diagram shown below, describe in set notation the region labeled "f".



Answers to 9.1 and 9.2

1. $X_1 = \begin{bmatrix} \frac{4}{15} \\ \frac{11}{15} \end{bmatrix}$

2. $X_2 = \begin{bmatrix} \frac{41}{150} \\ \frac{109}{150} \end{bmatrix}$

- 3.
- | | |
|-----------------------------|--------|
| <i>Business</i> | 34.17% |
| <i>Humanities</i> | 32.71% |
| <i>Engineering/Sciences</i> | 23.80% |
| <i>General Studies</i> | 9.31% |

- 4.
- | | |
|-----------------|--------|
| <i>AT&T</i> | 24.45% |
| <i>TMobile</i> | 28.45% |
| <i>Sprint</i> | 23.55% |
| <i>Verizon</i> | 23.55% |

- 5.
- a. regular
 - b. not regular
 - c. not regular
 - d. regular

6. $T^n = \begin{bmatrix} \frac{5}{16} & \frac{5}{16} \\ \frac{11}{16} & \frac{11}{16} \end{bmatrix}$

7. In ten years, 30.5% will be hybrid, 68.5% will be gasoline powered.

$$b. T^n = \begin{bmatrix} 0.3 & 0.3 \\ 0.7 & 0.7 \end{bmatrix}$$

In the long run, 30% will be hybrid and 70% gasoline powered.

8. *FoxSports* 33.75%
ESPN 44.34%
CBSSports 21.91%

8b. *FoxSports* 33.33%
ESPN 44.44%
CBSSports 22.22%

23. a. 0.2, b. 0.4, c. 0.8, d. 0.2

$$24. 0.2\overline{162} = \frac{8}{37}$$

$$25. y = 1.3496x - 2.6372$$

26. Corner Points $A = (0, 0), B = (0, 5), C = (4.29, 2.86), D = (6, 0)$

27. 151, 200

28. \$60

29. (2, 0)

30. $(B \cap C) \cap A^c$

Final Exam Review Answers

1. $(x, y, z) = (5, 2, 3)$

2. $X = \{1, 2, 3, \dots, 40\}$, finite discrete

3. $\sigma = 5.6576$

4. $250/700 = 5/14$

5. a. one solution, b. $(x, y, z) = (1, -6, 4)$

6. a. true, b. false, c. false, d. false

$$7. D = \begin{bmatrix} 10 & -13 \\ 3 & 4 \\ 4 & 3 \end{bmatrix}$$

b. 4

8. $y = -3$

9.	X	-2	0	2	4	6
	<i>freq</i>	15	12	5	10	8
	$P(X = x)$	0.3	0.24	0.1	0.2	0.16

9b. 0.46

10. 0.7986

11. \$33.45

12. 700 units

13. a. $\{a, b, c, e, f\}$, b. $\{c, e, f\}$

14. $\sqrt{34}$

15. 1

16. 3.29%

17. 53.12

18. 165

19. 2200

20. \$5400

21. \$172.50

22. 720