

Week in Review # 15

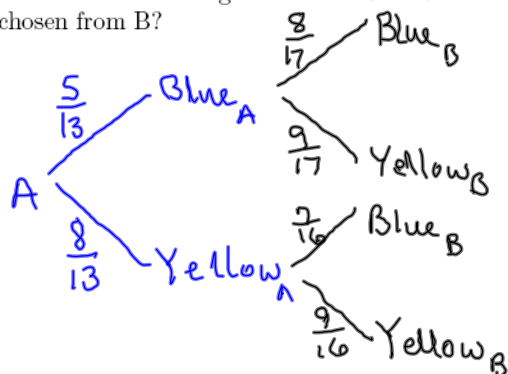
MATH 141

Drost-Spring 2010

Final Exam Review, Part B

1. Urn A contains 5 blue and 8 yellow balls. Urn B contains 7 blue and 9 yellow balls. A ball is chosen at random from Urn A. If it is blue, it is transferred to Urn B; otherwise, it is returned to Urn A. A ball is then chosen at random from Urn B.

What is the probability that a yellow ball was chosen from A given that a blue ball was chosen from B?



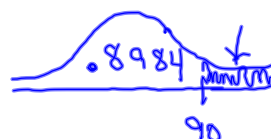
$$P(Y_A | B_B) = \frac{P(Y_A \cap B_B)}{P(B_B)}$$

$$= \frac{\frac{5}{13} \cdot \frac{8}{17} + \frac{8}{13} \cdot \frac{7}{16}}{\frac{5}{13} \cdot \frac{8}{17} + \frac{8}{13} \cdot \frac{7}{16}}$$

$$= 0.599989\dots$$

$$= 0.5980$$

2. A finite math class of 95 students takes a test. The test scores are normally distributed with a mean score of 76 and a standard deviation of 11. What is the probability that a randomly selected student scored a 90 or higher on the test?



$$P(X \geq 90) = 1 - (0.8984) = \boxed{0.1016}$$

$$P(X < 90) = P\left(Z < \frac{90 - \mu}{\sigma}\right)$$

$$= P\left(Z < \frac{90 - 76}{11}\right) = P\left(Z < \frac{14}{11}\right)$$

$$= 2^{\text{nd}} \text{ vars normalcdf}(-1E99, \frac{14}{11}, 0, 1)$$

$$= 0.8984425162$$

3. Carousel Paperbooks has 100 new titles they are considering showcasing in their front window. In how many ways can the store pick and arrange 30 of the new titles in the display area?

pick and arrange $\boxed{P(100, 30)}$

4. Use matrix multiplication to solve the following

$$x + y - 3z = 8$$

system of equations: $2x + 5y + z = -5$

$$x + 3y + 2z = 0$$

$$A = \begin{bmatrix} 1 & 1 & -3 \\ 2 & 5 & 1 \\ 1 & 3 & 2 \end{bmatrix} \quad B = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad C = \begin{bmatrix} 8 \\ -5 \\ 0 \end{bmatrix}$$

$$A \cdot B = C$$

$$A^{-1} \cdot A \cdot B = A^{-1} \cdot C$$

$$B = \begin{bmatrix} 11 \\ -49 \\ 18 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 7 & -11 & 16 \\ -3 & 5 & -7 \\ 1 & -2 & 3 \end{bmatrix}$$

5. Solve the following system of equations.

$$2x - y - z = 0$$

$$3x - 2y - z = -1$$

$$2y - 2z = 4$$

$$A = \begin{bmatrix} 2 & -1 & -1 & 0 \\ 3 & -2 & -1 & -1 \\ 0 & 2 & -2 & 4 \end{bmatrix}$$

matrix \rightarrow math \downarrow B: rref(A) Enter

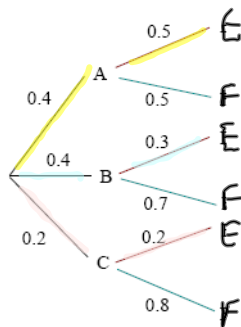
$$\begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

infinite number of solutions

$$\begin{aligned} y - z &= 2 & x - z &= 1 \\ y &= z + 2 & x &= z + 1 \end{aligned}$$

$$(x, y, z) = (z+1, z+2, z)$$

6. Given the following tree diagram, find $P(E)$. Round your answer to two decimal places.

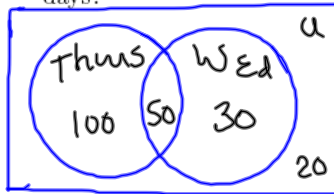


$$P(E) = 0.4(0.5) + 0.4(0.3) + 0.2(0.2)$$

$$P(E) = 0.20 + 0.12 + 0.04$$

$$P(E) = 0.36$$

7. Of 200 TV viewers, 150 watch TV on Thursdays, 80 watch TV on Wednesdays, and 50 watch TV on both Wednesdays and Thursdays. What is the probability that a randomly selected TV viewer will watch TV on exactly one of these days?



$$\frac{100 + 30}{200} = 0.65$$

8. I am trying to remember my cell phone number. I know the 3-digit prefix, and the four digits are 2, 4, 6 and 8, but I do not remember the order.

How many different options do I have for my number?

$$\underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} = 24 \text{ possibilities}$$

9. The frequency distribution of X is given below. What is the standard deviation of X , rounded to 4 decimal places?

| x | 10 | 12 | 14 | 18 | 25 |
|------|----|----|----|----|----|
| freq | 8 | 15 | 27 | 30 | 20 |

L_1 Sum = 100

$P(X=x)$ $\frac{8}{100}$ $\frac{15}{100}$ $\frac{27}{100}$ $\frac{30}{100}$ $\frac{20}{100}$ L_2

Stat edit

Stat \rightarrow Calc 1: 1-Var Stats L_1, L_2 Enter

$$\bar{x} = \mu = 16.78 \text{ mean}$$

$$\sigma_x = 4.8116$$

10. What monthly payment is required to amortize a loan of \$50,000 over 20 years if interest at the rate of 8% per year is charged on the unpaid balance and interest calculations are made at the end of each month.

TVM Solver

$$N = 20(12)$$

$$I\% = 8$$

$$PV = -50,000$$

$$PMT = \boxed{} = \$418.22$$

$$FV = 0$$

$$P/Y = C/Y = 12$$

11. Find the equation of the least-squares line of the

following data:

| | | | | | |
|---|---|---|----|----|----|
| x | 3 | 7 | 11 | 14 | 20 |
| y | 2 | 5 | 9 | 12 | 18 |

 L_1
 L_2

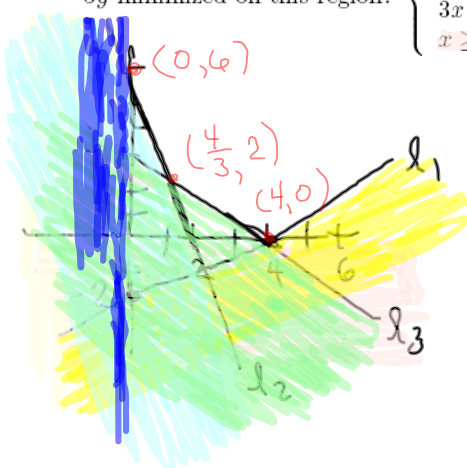
Stat edit L_1, L_2

Stat \rightarrow calc 4: Lin Reg L_1, L_2

$$y = 0.9529x - 1.2824$$

12. The following system of inequalities are constraints in a linear programming problem. Graph the feasible region, find the corner points. At what point is the objective function $f = 6x +$

5y minimized on this region?
$$\begin{cases} x - 2y \leq 4 \\ 3x + y \geq 6 \\ 3x + 4y \geq 12 \\ x \geq 0, y \geq 0 \end{cases}$$



$$f = 6x + 5y$$

$$f_A = 6(0) + 5(6) = 30$$

$$f_B = 6(\frac{4}{3}) + 5(2) = 18$$

$$f_C = 6(4) + 5(0) = 24$$

answer $(\frac{4}{3}, 2)$

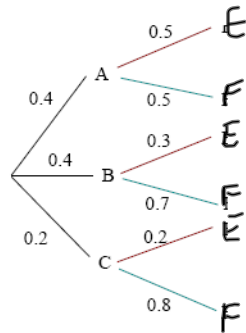
13. The tread life of a radial tire under normal conditions are normally distributed with a mean of 30,000 miles and a standard deviation of 2500 miles. What is the probability that a tire selected at random will have a tread life between 20,000 and 32,000 miles?

$$\text{normalcdf}(a, b, \mu, \sigma)$$

$$\text{normalcdf}(20000, 32000, 30000, 2500) =$$

$$\boxed{0.7881}$$

14. Given the following tree diagram, find $P(B|E)$. Round your answer to two decimal places.



$$P(B|E) = \frac{P(B \cap E)}{P(E)}$$

$$= \frac{0.4(0.3)}{0.4(0.5) + 0.4(0.3) + 0.2(0.2)}$$

$$= \frac{0.12}{0.20 + 0.12 + 0.04}$$

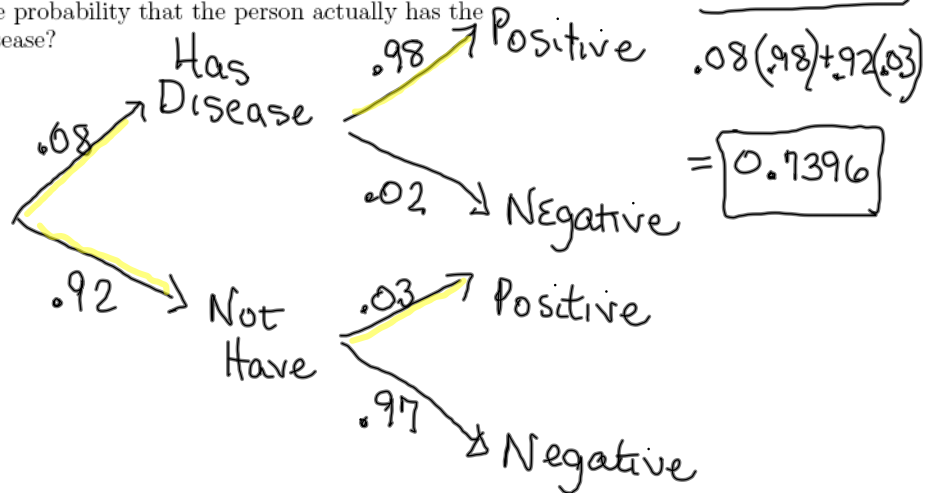
$$= \frac{0.12}{0.36} = \boxed{\frac{1}{3}}$$

15. An individual owns 25 identically-shaped plastic bowls, 5 red, 5 yellow, 5 green, 5 white and 5 blue. In how many distinct ways can they stack the bowls in a cupboard?

$$\frac{25!}{5! 5! 5! 5! 5!}$$

16. A medical test has been designed to detect the presence of a certain disease. Among those who have the disease, the probability that the disease will be detected by the test is 0.98. However, the probability that the test will give a false positive is 0.03. It has been estimated that 8% of the population has the disease. If the test administered to an individual is positive, what is

the probability that the person actually has the disease?



17. Back-yard Playgrounds sells forts for \$85 each. The manufacturer has weekly fixed costs of \$2350 and unit costs of \$40. What is the minimum number they must sell in order to not have a loss?

$$p = 85 \quad C = 2350 + 40x$$

$$f = 2350 \quad R = 85x$$

$$v = 40$$

$$P = R - C = 85x - (2350 + 40x)$$

$$P = 45x - 2350 = 0$$

$$45x = 2350$$

$$x = 52.2$$

53 forts

18. Given $A = \begin{bmatrix} 2 & -4 \\ 3 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 10 \\ 12 \end{bmatrix}$ What is the solution to $AX = B$

$$X = A^{-1} \cdot B$$

$$X = \begin{bmatrix} \frac{32}{7} \\ -\frac{3}{14} \end{bmatrix}$$

19. A binomial experiment consists of 10 independent trials. The probability of success in each trial is 0.22. What is the probability of obtaining at least 6 successes?

$$n = 10$$

$$p = 0.22$$

$$q = 0.78$$

$$P(X \geq 6) = P(6 \leq X \leq 10)$$

$$|- \text{binomcdf}(10, 0.22, 5)$$

$$|- 0.9896$$

$$\boxed{0.0104}$$

20.

| | Freshmen | Soph. | Junior | Senior |
|--------|----------|-------|--------|--------|
| Male | 75 | 35 | 20 | 10 |
| Female | 82 | 43 | 25 | 18 |

What is the probability that a student chosen at random from this group, is a freshman and a male?

$$\frac{75}{308} = \boxed{0.2435}$$

21. A box contains 5 blue pens, 2 red pens, 3 black pens, 4 purple markers, and 7 blue markers. In how many ways can Ellie pick 5 items, if she is to pick exactly 1 purple item and 2 blue items?

Blue Red Black Purple
 5 pens 2 pens 3 pens 4 markers
 7 markers

$$(12) \quad (2) \quad (3) \quad (4)$$

non blue
or
purple

1 Purple 2 Blue 2 (non blue/purple)

$$C(4, 1) C(12, 2) \cdot C(5, 2)$$

$$4 \cdot 66 \cdot 10 = \boxed{2640}$$

22. Find the inverse of matrix $A = \begin{bmatrix} 2 & -1 \\ 4 & 0 \end{bmatrix}$

$$A^{-1} = \begin{bmatrix} 0 & 1/4 \\ -1 & 1/2 \end{bmatrix}$$

23. Given the random variable Z is normally distributed, what is the probability that $-\frac{1}{2} \leq Z \leq \frac{1}{2}$?

$$\begin{aligned} P\left(-\frac{1}{2} \leq Z \leq \frac{1}{2}\right) &= \text{normalcdf}\left(-\frac{1}{2}, \frac{1}{2}, 0, 1\right) \\ &= \boxed{0.3829} \end{aligned}$$

24. You wish to purchase a \$25,000 new car. You put 10% down and finance the rest at 7.25% per year compounded monthly for 5 years. What are your monthly payments?

25000 cost of car
2500 down payment
22,500 loan

$$\begin{aligned} N &= 12(5) & FV &= 0 \\ I\% &= 7.25 & P/Y = C/Y &= 12 \end{aligned}$$

$$PV = -22500$$

$$PMT = \boxed{} = \$448.19$$

25. You roll a weighted six-sided die 100 times. The die is weighted such that the probability of the die showing a 5 is 0.6

$$n = 100$$

$$p = 0.6$$

$$q = 0.4$$

- a. What's the probability that exactly 25 five's are rolled?

$$P(X = 25) = \text{binompdf}(100, 0.6, 25)$$

$$= 9.8407 E^{-13}$$

- b. What's the probability that at least 45 fives are rolled?

$$P(X \geq 45) = P(45 \leq X \leq 100)$$

$$= 1 - \text{binomcdf}(100, 0.6, 44)$$

$$= 0.9991$$

- c. What's the probability that fewer than 60 fives are rolled?

$$P(X < 60) = P(0 \leq X \leq 59)$$

$$= \text{binomcdf}(100, 0.6, 59)$$

$$= 0.4567$$

- d. How many fives should you expect to roll?

$$E(X) = \text{since binomial} = np = 100(0.6)$$

$$= 60$$

- e. What is the variance and standard deviation in the number of fives rolled?

$$\text{since binomial} \quad \text{var}(X) = npq = 100(0.6)(0.4)$$

$$= 24$$

$$\text{Standard deviation} =$$

$$\sigma_X = \sqrt{npq}$$

$$\sigma_X = 4.8990$$