Week in Review

MATH 141
1.5,2.1,2.4,2.5

Spring 2010

#1. Stat, Edit, input data in L_1, L_2
Stat, Calc, 4: LinReg (ax+b) L_1, L_2, Y_1 (E)

1. Find a straight line which best models the following data:

$y_1 = 0.6982710541x + 14.45398773$
$r = 0.820539771$

Birth Weight (lbs)	7.5	9.4	9.1	6.75	7.1	5.8
ength (inches)	19.5	21.5	20	19	20.5	18.1

L_1
L_2

a) Use the result to estimate the length of a newborn who weighed 9lb 4ozs. if $x = 9.25$ $y = 20.9$ in

b) Is this a good estimate? why or why not?

c) According to this model, what happens when the length increases 1 inch?

a. 2nd Calc 1: value
$x = 9.25$ enter
$y = 20.9$ inches

b. Yes, because the corr. coef (r) is close to 1 (or −1)
$r \approx 0.82$

c. $y = 21.9$ what is x?
$y_1 = $ LinReg
$y_2 = 21.9$
} open window so you see the lines intersect
$x = 10.7$ lbs

2. The following data relates x, annual miles driven in thousands, and y, the cost per mile in cents, of operating a new car.

Stat, Edit

x	5	10	15	20	25	30	35
y	50.3	34.8	30.1	27.4	25.6	23.5	21.8

L_1
L_2

a) assuming a linear relationship, find the best-fitting line. Stat, → Calc, 4: LinReg (ax+b) L_1, L_2, Y_1

b) use the results to estimate the cost per mile of operating a new company car if it is driven 12,000 miles during the firsst year of ownership.

a. $y = -0.8043x + 46.5857$

b. 12,000 miles $\Rightarrow x = 12$
2nd Calc, 1: value, $x = 12$ Enter $y = $ 36.9 cents/mile

3. The median price of homes in thousands of dollars in a certain city from 1985 to 2000 was:

Year	1985	1988	1992	1995	1998	2000
Price	68.2	70.2	76.1	79.5	85.1	89.9

rewrite L_1 in terms of 1985 → $t=0$

a) Using 1985 as t=0, and assuming a linear relationship, find the equation which best fits the data provided.

b) Using this model, predict the median price for the year 2003, to the nearest thousand.

L_1: 0, 3, 7, 10, 13, 15
L_2: 68.2, 70.2, 76.1, 79.5, 85.1, 89.9
Stat → Calc 4: LinReg L_1, L_2, Y_1
a. $y = 1.4351x + 66.6857$ $r = 0.9878$ ✓ very good

b) 2003 → $x = 18$
2nd calc, 1. value $x = 18$ Enter
$y = 92.517857$
\$93,000

4. Rudder Theatre has a seating capacity of 1000 people, and charges \$5 for children, \$8 for students and \$12 for adults. At a recent function with full attendance, there were sixteen times as many students as children. The number of students was four times the number of adults and children combined. Write a system of equations which describes the data.

s = # students
c = # children
a = # adults

$$\begin{cases} s + c + a = 1000 \\ s = 16c \\ s = 4(a + c) \end{cases}$$

$$\begin{cases} s + c + a = 1000 \\ s - 16c = 0 \\ s - 4c - 4a = 0 \end{cases}$$

5. Samuel ones has \$20,000 to invest in two accounts, with account A yielding $8\frac{1}{2}\%$ and account B yielding 11.25%. Write a system of equations given that the annual interest earned is \$1810.

A = \$ invested in acct A
B = \$ invested in acct B

$$\begin{cases} A + B = 20{,}000 \\ 0.085A + 0.1125B = 1810 \end{cases}$$

6. Determine whether the following systems have only one solution, infinitely many solutions, or no solutions. Find all solutions when they exist.

a)
$$\frac{5}{4}x - \frac{2}{3}y = 16$$
$$\frac{1}{4}x - \frac{5}{3}y = -4$$

$$\left[\begin{array}{cc|c} \frac{5}{4} & -\frac{2}{3} & 16 \\ \frac{1}{4} & -\frac{5}{3} & -4 \end{array}\right] \xrightarrow[4\cdot R_2]{\frac{4}{5}\cdot R_1} \left[\begin{array}{ccc} 1 & -\frac{8}{15} & \frac{64}{5} \\ 1 & -\frac{20}{3} & -16 \end{array}\right] \xrightarrow{-1\cdot R_1 + R_2} \left[\begin{array}{ccc} 1 & -\frac{8}{15} & \frac{64}{5} \\ 0 & -\frac{92}{15} & -\frac{144}{5} \end{array}\right]$$

$$\xrightarrow{-\frac{15}{92}\cdot R_2} \left[\begin{array}{ccc} 1 & -\frac{8}{15} & \frac{64}{5} \\ 0 & 1 & \frac{108}{23} \end{array}\right] \xrightarrow{\frac{8}{15}R_2 + R_1} \left[\begin{array}{ccc} 1 & 0 & \frac{352}{23} \\ 0 & 1 & \frac{108}{23} \end{array}\right]$$

$x = \frac{352}{23}$

$y = \frac{108}{23}$

Quick check

Matrix A = $\left[\begin{array}{ccc} 5/4 & -2/3 & 16 \\ 1/4 & -5/3 & -4 \end{array}\right]$

matrix → math

b. rref(A) Enter

b)
$$5x + 7.5y = -30$$
$$-2x - 3y = 12$$

$$\left[\begin{array}{cc|c} 5 & 7.5 & -30 \\ -2 & -3 & 12 \end{array}\right] \xrightarrow{\frac{1}{5}R_1} \left[\begin{array}{ccc} 1 & 1.5 & -6 \\ -2 & -3 & 12 \end{array}\right]$$

$$\xrightarrow{2R_1 + R_2} \left[\begin{array}{ccc} 1 & 1.5 & -6 \\ 0 & 0 & 0 \end{array}\right]$$

infinitely many solutions

$x + \frac{3}{2}y = -6$

$\frac{3}{2}y = -x - 6$

$y = -\frac{2}{3}x - 4$

let $x = t$

$y = -\frac{2}{3}t - 4$

$\boxed{(t, -\frac{2}{3}t - 4)}$

7. Find the value(s) of for which the following system has:

a) no solution

$$\begin{cases} 3x - y = 12 \\ 6x - 2y = \end{cases}$$

$$\left[\begin{array}{ccc} 3 & -1 & 12 \\ 6 & -2 & K \end{array}\right] \xrightarrow{-2R_1 + R_2} \left[\begin{array}{ccc} 3 & -1 & 12 \\ 0 & 0 & -24 + K \end{array}\right]$$

no solution when

$-24 + K \neq 0$

$\boxed{K \neq 24}$

b) an infinite number of solution

$$\begin{cases} 3x - y = 12 \\ 2x + y = 8 \end{cases}$$

$$\left[\begin{array}{ccc} 3 & -1 & 12 \\ 2 & K & 8 \end{array}\right] \xrightarrow[-3R_2]{2R_1} \left[\begin{array}{ccc} 6 & -2 & 24 \\ -6 & -3K & -24 \end{array}\right] \xrightarrow{R_1 + R_2} \left[\begin{array}{ccc} 6 & -2 & 24 \\ 0 & -2-3K & 0 \end{array}\right]$$

infinite # of solutions when $-2 - 3K = 0$

$-3K = 2$

$K = \boxed{\frac{-2}{3}}$

8. a. Write the following system of equations as an augmented matrix.

$$\begin{aligned} 2x + 3y + 5 &= 6 \\ 3x - y - 3 &= 11 \\ x + y + &= 1 \end{aligned}$$

a. $\left[\begin{array}{ccc|c} 2 & 3 & 5 & 6 \\ 3 & -1 & -3 & 11 \\ 1 & 1 & 1 & 1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 3 & -1 & -3 & 11 \\ 2 & 3 & 5 & 6 \end{array}\right]$

b. Solve the system using Gauss- ordan elimination method.

$\xrightarrow[-2R_1+R_3]{-3R_1+R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -4 & -6 & 8 \\ 0 & 1 & 3 & 4 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 4 \\ 0 & -4 & -6 & 8 \end{array}\right] \xrightarrow{4R_2+R_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 6 & 24 \end{array}\right] \xrightarrow{\frac{1}{6}R_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 1 & 4 \end{array}\right] \xrightarrow{-3R_3+R_2}$

$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & -8 \\ 0 & 0 & 1 & 4 \end{array}\right] \xrightarrow{-1R_2+R_1} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 9 \\ 0 & 1 & 0 & -8 \\ 0 & 0 & 1 & 4 \end{array}\right] \xrightarrow{-1R_3+R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -8 \\ 0 & 0 & 1 & 4 \end{array}\right]$

$(x, y, z) = (5, -8, 4)$

9. Solve problem 4 using Gauss- ordan elimination method.

$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1{,}000 \\ -16 & 1 & 0 & 0 \\ -4 & 1 & -4 & 0 \end{array}\right] \xrightarrow[4R_1+R_3]{16R_1+R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1{,}000 \\ 0 & 17 & 16 & 16{,}000 \\ 0 & 5 & 0 & 4{,}000 \end{array}\right] \xrightarrow[\frac{1}{5}R_3]{} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1{,}000 \\ 0 & 17 & 16 & 16{,}000 \\ 0 & 1 & 0 & 800 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3}$

$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1{,}000 \\ 0 & 1 & 0 & 800 \\ 0 & 17 & 16 & 16{,}000 \end{array}\right] \xrightarrow[-17R_2+R_3]{-1R_2+R_1} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 200 \\ 0 & 1 & 0 & 800 \\ 0 & 0 & 16 & 2400 \end{array}\right] \xrightarrow[\frac{1}{16}R_3]{} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 200 \\ 0 & 1 & 0 & 800 \\ 0 & 0 & 1 & 150 \end{array}\right] \xrightarrow{-1R_3+R} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 50 \\ 0 & 1 & 0 & 800 \\ 0 & 0 & 1 & 150 \end{array}\right]$

50 children
800 students 150 adults

10. a. Write the following system of equations as an augmented matrix.

$$\begin{aligned} 4x - 4 &= 0 \\ 2y - 4 &= 6 \\ 2x - y &= -3 \end{aligned}$$

$\left[\begin{array}{ccc|c} 4 & 0 & -4 & 0 \\ 0 & 2 & -4 & 6 \\ 2 & -1 & 0 & -3 \end{array}\right]$

b. Solve the system using Gauss- ordan elimination method. If the system has an infinite number of solutions, write three possible solutions.

$\xrightarrow[\frac{1}{2}R_2]{\frac{1}{4}R_1} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & -2 & 3 \\ 2 & -1 & 0 & -3 \end{array}\right] \xrightarrow[-2R_1+R_3]{} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & -2 & 3 \\ 0 & -1 & 2 & -3 \end{array}\right] \xrightarrow[R_2+R_3]{} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right] \leftarrow$ infinite number of solutions

$y - 2z = 3$
$y = 2z + 3$

$x - z = 0$
$x = z$

let $x = t$
$x = t$
$y = 2t + 3$
$z = t$

$(t, -2t+3, t)$
$(1, 1, 1)$
$(2, -1, 2)$
$(3, -3, 3)$

11. a. Write the following system of equations as an augmented matrix.

$$\begin{aligned} 2x + y &= 12 \\ 3x - y &= 10 \\ x + y &= -2 \end{aligned}$$

$$\left[\begin{array}{cc|c} 2 & 1 & 12 \\ 3 & -1 & 10 \\ 1 & 1 & -2 \end{array}\right]$$

b. Solve the system using Gauss- ordan elimination method. If the system has an infinite number of solutions, write three possible solutions.

$$\xrightarrow{-R_3+R_1} \left[\begin{array}{cc|c} 1 & 0 & 14 \\ 3 & -1 & 10 \\ 1 & 1 & -2 \end{array}\right] \xrightarrow[-R_1+R_3]{-3R_1+R_2} \left[\begin{array}{cc|c} 1 & 0 & 14 \\ 0 & -1 & -32 \\ 0 & 1 & -16 \end{array}\right] \xrightarrow{-1\cdot R_2} \left[\begin{array}{cc|c} 1 & 0 & 14 \\ 0 & 1 & 32 \\ 0 & 1 & -16 \end{array}\right] \xrightarrow{-1\cdot R_2+R_3}$$

$$\left[\begin{array}{cc|c} 1 & 0 & 14 \\ 0 & 1 & 32 \\ 0 & 0 & -48 \end{array}\right] \leftarrow \text{no solution}$$

Given matrix $A = \begin{bmatrix} 2 & 3 & -1 \\ 4 & 5 & 6 \end{bmatrix}$, $B = \begin{matrix} 1 & 2 \\ 3 & -1 \\ 0 & 4 \end{matrix}$, $C = \begin{bmatrix} -1 & 1 & 2 \\ 0 & -4 & -3 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 2 & \triangle \\ 4 & -2 & 0 \end{bmatrix}$

12. Find A - C $= \begin{bmatrix} 2 & 3 & -1 \\ 4 & 5 & 6 \end{bmatrix} - \begin{bmatrix} -1 & 1 & 2 \\ 0 & -4 & -3 \end{bmatrix} = \begin{bmatrix} 2-(-1) & 3-(1) & -1-(2) \\ 4-(0) & 5-(-4) & 6-(-3) \end{bmatrix}$

$$= \begin{bmatrix} 3 & 2 & -3 \\ 4 & 9 & 9 \end{bmatrix}$$

13. Find A + C =

$$\begin{bmatrix} 2 & 3 & -1 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} -1 & 1 & 2 \\ 0 & -4 & -3 \end{bmatrix} = \begin{bmatrix} 2+(-1) & 3+(1) & -1+(2) \\ 4+(0) & 5+(-4) & 6+(-3) \end{bmatrix} = \begin{bmatrix} 1 & 4 & 1 \\ 4 & 1 & 3 \end{bmatrix}$$

14. Find A^T

T transpose $R_1 \to C_1$, $R_2 \to C_2$

$$A = \begin{bmatrix} 2 & 3 & -1 \\ 4 & 5 & 6 \end{bmatrix} \qquad A^T = \begin{bmatrix} 2 & 4 \\ 3 & 5 \\ -1 & 6 \end{bmatrix}$$

15. Find $A \cdot B$

$$\underset{2\times3}{A} = \begin{bmatrix} 2 & 3 & -1 \\ 4 & 5 & 6 \end{bmatrix} \quad \underset{3\times2}{B} = \begin{bmatrix} 1 & 2 \\ 3 & -1 \\ 0 & 4 \end{bmatrix}$$

$A \cdot B_{2\times2}$

$$\underset{2\times2}{A \cdot B} = \begin{bmatrix} r_1 \cdot c_1 & r_1 \cdot c_2 \\ r_2 \cdot c_1 & r_2 c_2 \end{bmatrix} = \begin{bmatrix} 2+9+0 & 4-3-4 \\ 4+15+0 & 8-5+24 \end{bmatrix} = \begin{bmatrix} 11 & -3 \\ 19 & 27 \end{bmatrix}$$

16. Find $D \cdot B$

$$\underset{2\times3}{D} = \begin{bmatrix} 1 & 2 & \triangle \\ 4 & -2 & 0 \end{bmatrix} \quad \underset{3\times2}{B} = \begin{bmatrix} 1 & 2 \\ 3 & -1 \\ 0 & 4 \end{bmatrix}$$

$$D \cdot B = \begin{bmatrix} r_1 c_1 & r_1 c_2 \\ r_2 c_1 & r_2 c_2 \end{bmatrix}$$

$$\underset{2\times2}{D \cdot B} = \begin{bmatrix} 1+6+0 & 2-2+4\triangle \\ 4-6+0 & 8+2+0 \end{bmatrix} = \begin{bmatrix} 7 & 4\triangle \\ -2 & 10 \end{bmatrix}$$

17. rders were received for the following: 40 regular burritos on wheat, 50 monster burritos on cayenne, and 36 regular burritos on spinach.

The order will be filled at three locations as outlined below.

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location	reg/wheat	monster/cayene	reg/spinach
I	20	15	10
II	15	10	20
III	5	25	6

The ingredients needed for the various orders are as outlined below: (units are cups except for the tortillas)

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ingredients	rice	cheese	wheat	cayene	spinach	chicken	tomatoes
regular/wheat	.5	.5	1	0	0	.5	.5
monster/cayene	1	.75	0	1	0	1	1
regular/spinach	.5	.5	0	0	1	.5	.5

The prices of the various ingredients per unit is listed below.

$P =$

ingregients	\$/serving
rice	.25
cheese	.50
wheat tortilla	.05
cayene tortilla	.08
spinach tortilla	.09
chicken	1.25
tomatoes	.30

abel the matricies (for order), I (for ingredients), and P (for prices), and answer the following questions:

a) find $\cdot$ and explain what it represents.

b) find $(\cdot) \cdot P$ and explain what it represents.

Given the following prices for each item on the menu, build another matrix, and label it D (for dollars):

$D =$

regular/wheat	4.25
monster/cayene	5.00
regular/spinach	4.25

c) find $\cdot D$ and explain what it represents.

d) find $\cdot D - (\cdot) \cdot P$ and explain what it represents.

a. $O \cdot I = \begin{bmatrix} 20 & 15 & 10 \\ 15 & 10 & 20 \\ 5 & 25 & 6 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 1 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & 3/4 & 0 & 1 & 0 & 1 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

3×3 3×7

using calculator

Enter O in matrix A

I in matrix B

$O \cdot I = A \cdot B$

rice cheese wheat cay. spin chicken tomatoes

$O \cdot I_{3\times 7} = \begin{bmatrix} 30 & 26.25 & 20 & 15 & 10 & 30 & 30 \\ 27.5 & 25 & 15 & 10 & 20 & 27.5 & 27.5 \\ 30.5 & 24.25 & 5 & 25 & 6 & 30.5 & 30.5 \end{bmatrix}$

row1 is location1 amt of each ingred needed to fill order

row2 is location2

row3 is location3

b) $(O \cdot I) \cdot P = \begin{bmatrix} 70.225 \\ 65.35 \\ 69.815 \end{bmatrix}$ Cost at each location to fill order

3×7 7×1

3×1

c) $O \cdot D = \begin{bmatrix} 202.50 \\ 198.75 \\ 171.75 \end{bmatrix}$ revenue coming IN at each location

3×3 3×1

3×1

d) $O \cdot D - (O \cdot I) \cdot P =$

$\begin{bmatrix} 132.275 \\ 133.40 \\ 101.935 \end{bmatrix}$ profit at Each location on the orders

18. If $A \cdot B = 0$, where A and B are matricies, is it true that A = 0 or B = 0 ?

no, for Example

$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 4 & 6 \\ -2 & -3 \end{bmatrix}$ $A \cdot B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

19. Find A such that:

$$A \cdot \begin{bmatrix} 2 & 1 \\ -3 & 0 \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ -6 & 3 \end{bmatrix}$$

$A \cdot \begin{bmatrix} 2 & 1 \\ -3 & 0 \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ -6 & 3 \end{bmatrix}$ $A = \begin{bmatrix} w & x \\ y & z \end{bmatrix}$

$\begin{bmatrix} w & x \\ y & z \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ -3 & 0 \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ -6 & 3 \end{bmatrix}$

$\begin{bmatrix} 2w-3x & w \\ 2y-3z & y \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ -6 & 3 \end{bmatrix}$

$\begin{cases} 2w-3x=-4 \\ w=1 \\ 2y-3z=-6 \\ y=3 \end{cases}$

Since $w=1$: $2(1)-3x=-4$, $-3x=-6$, $x=2$

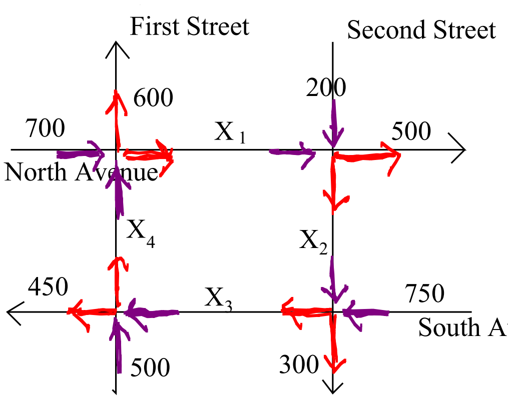
Since $y=3$: $2(3)-3z=-6$, $-3z=-12$, $z=4$

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

20. The figure below shows the traffic flow around Park Central. The arrows indicate the direction of traffic flow on each one-way road, and the average number of vehicles per hour entering and leaving each intersection appears beside each road.

a. Write a general expression for traffic flow, in terms of x_1, x_2, x_3, and x .

b. Find a possible flow pattern that will not cause congestion.



$\begin{cases} 700+x_4=600+x_1 \\ 200+x_1=500+x_2 \\ 750+x_2=300+x_3 \\ 500+x_3=450+x_4 \end{cases}$ $\begin{cases} x_1-x_4=100 \\ x_1-x_2=300 \\ x_2-x_3=-450 \\ x_3-x_4=-50 \end{cases}$

$x_1\ x_2\ x_3\ x_4\ N$

$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 100 \\ 1 & -1 & 0 & 0 & 300 \\ 0 & 1 & -1 & 0 & -450 \\ 0 & 0 & 1 & -1 & -50 \end{array}\right] \xrightarrow{-R_1+R_2} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 100 \\ 0 & -1 & 0 & 1 & 200 \\ 0 & 1 & -1 & 0 & -450 \\ 0 & 0 & 1 & -1 & -50 \end{array}\right] \xrightarrow[-1 \cdot R_2]{R_2+R_3} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 100 \\ 0 & 1 & 0 & -1 & -200 \\ 0 & 0 & -1 & 1 & -250 \\ 0 & 0 & 1 & -1 & -50 \end{array}\right]$

$\xrightarrow{R_3+R_4} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 100 \\ 0 & 1 & 0 & -1 & -200 \\ 0 & 0 & -1 & 1 & -250 \\ 0 & 0 & 0 & 0 & -300 \end{array}\right]$ no Solution!