Week in Review # 6

MATH 141 Drost-Spring 2010

6.1, 6.2, 6.3, 6.4

1. Write each set with roster notation:

   a) $A = \{x | x \in I, 2x - 4 > 3\}$

   b) $B = \{x|$ x is a state west of California $\}$

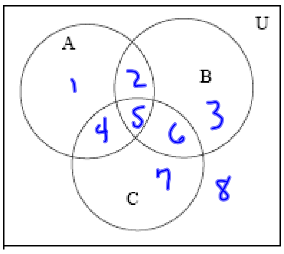
a) $A = \{4, 5, 6, 7, \ldots\}$ b) $B = \{\text{Hawaii}, \text{Alaska}\}$

$2x - 4 > 3$

$2x > 7$

$x > 3.5$

2. Shade the region described below:



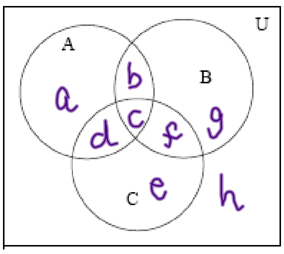
a) $A \cap (B \cup C)$

$A = \{1, 2, 4, 5\}$ $B = \{2, 3, 5, 6\}$

$C = \{4, 5, 6, 7\}$

$B \cup C = \{2, 3, 4, 5, 6, 7\}$

$A \cap (B \cup C) = \{2, 4, 5\}$

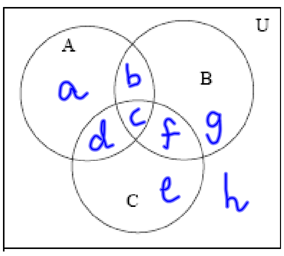


b) $B^c \cap A$

$B = \{b, c, f, g\}$

$A = \{a, b, c, d\}$ $B^c = \{a, d, e, h\}$

$B^c \cap A = \{a, d\}$



c) $(A \cap B \cap C)^c$

$(A \cap B \cap C) = \{c\}$

$(A \cap B \cap C)^c = \{a, b, d, e, f, g, h\}$

3. Describe each of the following:

$U = \{\text{students enrolled in this class}\}$
$A = \{\text{students who play bridge}\}$
$B = \{\text{students who live in dorms}\}$
$C = \{\text{students who drive an SUV}\}$

a) $A \cap C$ students who play bridge and drive an SUV

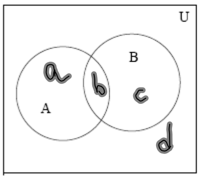
b) $B^c \cup A$

c) $C^c \cap B$

b) $B^c = \{\text{students who do NOT live in dorms}\}$
$A = \{\text{students who play bridge}\}$
$B^c \cup A = \{\text{students who do not live in dorms or play bridge}\}$

c) $C^c = \{\text{students who do NOT drive an SUV}\}$
$C^c \cap B = \{\text{students who do NOT drive an SUV and play bridge}\}$

4. Given A and B are subsets of U,
$n(U) = 250, n(A) = 90, n(B) = 165,$
$n(A \cap B) = 25$, find each of the following.

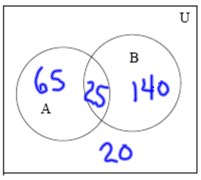


$b = 25$
$a + b = 90$
$a + 25 = 90$
$a = 65$

$b + c = 165$
$25 + c = 165$
$c = 140$
$a + b + c + d = 250$
$65 + 25 + 140 + d = 250$
$230 + d = 250$
$d = 20$

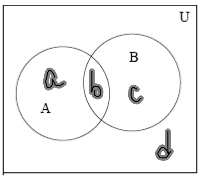
a) $n(A \cup B)$,

$a + b + c =$
$65 + 25 + 140 = \boxed{230}$



$n(A) = 65 + 25 = 90$
$n(A^c) = 250 - 90 = \boxed{160}$

b) $n(A^c)$



c) $n(A^c \cup B^c)$

$A = \{a, b\}$ $A^c = \{c, d\}$
$B = \{b, c\}$ $B^c = \{a, d\}$
$A^c \cup B^c = \{a, c, d\}$
$n(A^c \cup B^c) = 65 + 140 + 20$
$= \boxed{225}$

5. A poll of 200 shoppers found the following:

60 carried an American Express card

60 carried a Bank Americard

95 had Aggie Bucks

55 had at least two of these

20 had Bank Americard and Aggie Bucks

125 had American Express or Aggie Bucks

140 did not have American Express

105 had American Express or Bank Americard

$a+b+c+d=60$

$b+c+f+g=60$

$d+c+f+e=95$

$b+c+d+f=55$

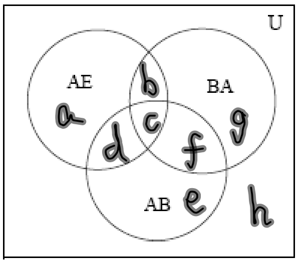
$c+f=20$

$a+b+c+d+e+f=125$

$e+f+g+h=140$

$a+b+c+d+f+g=105$

$a+b+c+d+e+f+g+h=200$



a) how many shoppers carried all three cards? 5

b) how many had none of the three? 45

c) how many had only one card?

$a+g+e=\boxed{100}$

matrix A 9x9, rref(A)

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 20 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 10 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 25 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 50 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 15 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 30 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 45 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$a=20$

$b=10$

$c=5$

$d=25$

$e=50$

$f=15$

$g=30$

$h=45$

6. A box contains five biographies, four mysteries, and six children's books.

a) How many ways can these books be arranged on a shelf?

b) How many ways can these books be arranged if the biographies, mysteries and children's books are kept together?

a) total of 15 books $15(14)(13)(12)\ldots(3)(2)(1) = \boxed{15!}$

b) Choose type · Biog · Myst. · Child.

$P(3,3) \cdot P(5,5) \cdot P(4,4) \cdot P(6,6)$

$6 \cdot 120 \cdot 24 \cdot 720 = \boxed{12{,}441{,}600}$

7. How many different committees can be formed if it is composed of two faculty members and ten students, given that five faculty have applied and thirty students applied?

$C(5,2) \cdot C(30,10)$

$10\,(30045015)$

$\boxed{300{,}450{,}150}$

8. How many different arrangements are possible with the letters of the word: S U C C E S S

$\frac{7!}{(3!)(2!)} = \frac{5040}{(6)(2)} = \boxed{420}$

9. A box contains two red marbles, three blue marbles, and five black marbles. How many ways can you select one red, two blue and three black marbles? Each of the marbles has a different letter of the alphabet painted on its surface.

R R

B B B

bl bl bl bl bl

$C(2,1) \cdot C(3,2) \cdot C(5,3)$

$2 \cdot 3 \cdot 10 = 60$

10. A shipment arrives of 500 batteries, 20 of which are defective. A sample of ten is selected at random.

20 defective

480 not-defective

a) In how many different ways can the samples be selected?

$C(500,10) = 2.458105888 \times 10^{20}$

b) How many samples contain six defective batteries?

$C(20,6) \cdot C(480,4)$

$(38760) \cdot (2184297480)$

$8.466337032 \times 10^{13}$

c) How many samples contain no defective batteries?

$C(20,0) \cdot C(480,10)$

$1 \cdot 1.628032027 \times 10^{20}$

$1.628032027 \times 10^{20}$

11. Given $U = \{1, 2, 3, 4, 5\}$, $A = \{2, 4, 5\}$, and $B = \{4, 5\}$

Label each of the following as True or false:

a. $4 \subseteq A$ false

an Element cannot be a "subset of"

b. $B \subseteq A$ true

Every Element in B is also in A

c. $\phi \subseteq A$ true

null set is a subset of every set

d. $5 \in A$ true

5 is an Element of set A

e. $5 \in B^C$

$B = \{4, 5\}$

false $B^c = \{1, 2, 3\}$

Find the elements in each of the following:

f. $A \cup B$

$A = \{2, 4, 5\}$

$B = \{4, 5\}$

$A \cup B = \{2, 4, 5\}$

g. $A \cup B^C$ $= \{1, 2, 3, 4, 5\}$

$A = \{2, 4, 5\}$

$B^c = \{1, 2, 3\}$

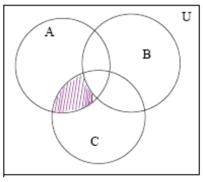
h. $A^C \cap B^C$ $= \{1, 3\}$

$A^c = \{1, 3\}$

$B^c = \{1, 2, 3\}$

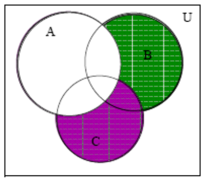
Describe the shaded regions below: answers are not unique

i.



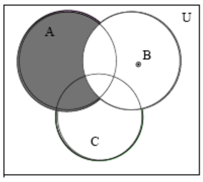
$(A \cap C) \cap B^c$

j.



$(B \cup C) \cap A^c$

k.



$A \cap B^c$

12. Given $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $A = \{2, 4, 6, 8\}$, $B = \{1, 2, 3, 4\}$

number of subsets $= 2^n$

number of proper subsets $= 2^n - 1$

a. list all the proper subsets of A

b. find $n(A \cap B^C)$

c. true or false: A = B

d. true or false: n(A) = n(B)

e. find: $N(A^C \cap B)$

a. $\{\ \}, \{2\}, \{4\}, \{6\}, \{8\}$
$\{2,4\}, \{2,6\}, \{2,8\}, \{4,6\}, \{4,8\}$
$\{6,8\}, \{2,4,6\}, \{2,4,8\}, \{4,6,8\}$
$\{2,4,6,8\}, \{2,6,8\}$

b. $A = \{2,4,6,8\}$
$B^c = \{5,6,7,8\}$
$A \cap B^c = \{6,8\}$
$n(A \cap B^c) = 2$

c. false do not have same elements

d. true they both have 4 elements

e. $A^c = \{1,3,5,7\}$ $B = \{1,2,3,4\}$

$A^c \cap B = \{1,3\}$

$n(A^c \cap B) = 2$

13. Of the 50 students at Midtown School, there were:

27 who played basketball

19 who played volleyball

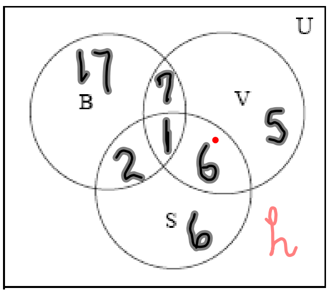
15 who played soccer

8 who played basketball and volleyball

7 who played volleyball and soccer

2 who played basketball and soccer only

1 who played volleyball, basketball and soccer.



a) How many played only volleyball and soccer? 6

b) How many played only one sport? $17+5+6=28$

c) How many played at least two sports? $7+1+2+6=16$

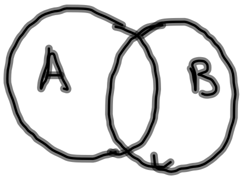
d) How many did not play in these sports?

$$17+7+1+2+6+6+5+h=50$$

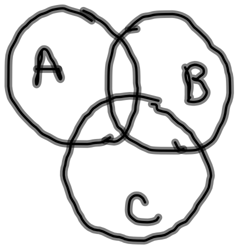
$$44+h=50$$

$$h=6$$

Useful formulas:



$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$



$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$$