

Math 142 Exam I Review

★ Slope:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{-3}{2} \text{ means as } x \text{ increases 2 units, } y \text{ decreases 3 units}$$

Horizontal lines: $m = 0$

Vertical lines: **slope is undefined**

★ Linear Functions: straight lines

1. $C = 2.5x + 1000$

Since $C(0) = 1000$, the fixed costs are 1000.

The variable costs are 2.5 per item.

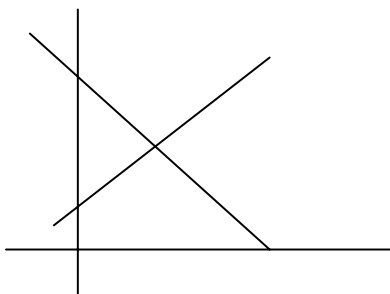
2. If given two pair of points and asked for a linear function

input data in L_1 and L_2

change window to match data, resetting the x – data to L_1 , and the y- data to L_2

stat, -> to calculate, 4:LinReg L_1, L_2, y_1 enter

3. supply and demand



Supply equations: positive slope

Demand equations: negative slope

Intersection: Equilibrium point (x, p)
 x is the *equilibrium quantity* and
 p is the *equilibrium price*

★ Functions: one x is always paired with exactly 1 y response.

Domain: possible values for x is all reals unless

a) fraction : check to see if any x values make the denominator 0.

b) even-index radical $\sqrt[a]{M}$, if the root a is even, then M must be non-negative $M \geq 0$

c) logarithm: $\log(a)$, $a > 0$ the argument must be positive.

d) word problems – only values of x which make sense are allowed.

Vertical line test for functions

Horizontal line test for 1-1

A function must be 1-1 to have an inverse.

Find the inverse by switching all x 's and y 's and solve for the new y ; rename y , $f^{-1}(x)$

Application Problems

$$R = x \bullet p$$

$$P = R - C$$

Vertical and Horizontal shifts

$$f(x) = (x - a)^5 + 6$$

rigid transformation of the graph of $y = x^5$, shifted **a** units right, and **b** units up.

Reflections

$$f(x) = -x^5$$

the graph of $y = x^5$ reflected about the **x**-axis

Stretches and Shrinks

$f(x) = 6x^5$ similar to the graph of $y = x^5$, stretched vertically by a factor of 6

$f(x) = \frac{1}{2}x^5$ similar to the graph of $y = x^5$, vertical shrink by a factor of $\frac{1}{2}$

Combinations: shifts up or down must occur **AFTER** reflections about the x-axis.

★ Quadratic Functions:

Standard form: $y = ax^2 + bx + c$, $a \neq 0$

Vertex form: $y = a(x - h)^2 + k$ where **(h, k)** is the vertex.

Where is the vertex?

Where is the axis of symmetry?

What is the maximum value?

Find the vertex of $f(x) = -5x^2 - 40x + 300$

A) By formula $h = \frac{-b}{2a}$ $h = \frac{-(-40)}{2(-5)} = -4$

and $k = f(h) = -5(-4)^2 - 40(-4) + 300 = 380$

so the vertex is at **(-4, 380)**

B) By completing the square:

$$f(x) = -5x^2 - 40x + 300$$

$$f(x) = -5(x^2 + 8x) + 300$$

$$f(x) = -5(x^2 + 8x + 16) + 300 + 80$$

$$f(x) = -5(x + 4)^2 + 380$$

therefore the vertex is at **(-4, 380)**

C) by calculator

$$y_1 = -5x^2 - 40x + 300, 2^{\text{nd}} \text{ calc maximum}$$

Note: this is often an approximate answer and not exact!

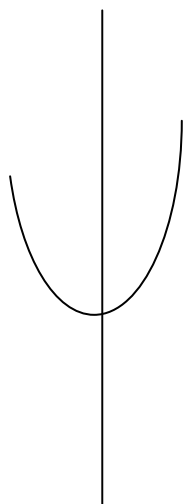
Quadratic formula

$$\text{If } ax^2 + bx + c = 0, a \neq 0, \text{ then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ if } b^2 - 4ac \geq 0.$$

★ Intercepts:

Substitute in $x = 0$, and solve for y

Substitute in $y = 0$, and solve for x .



Line of symmetry passes through the vertex and has the equation, $x = h$

Max or Min value is k depending on if the parabola opens up or down.

★ Break-Even Analysis: $R = C$

★ Polynomial Functions:

$y = ax^n + bx^{n-1} + \dots + z$ where all the exponents are non-negative integers

★ Rational Functions:

$f(x) = \frac{n(x)}{d(x)}$ where $n(x)$ and $d(x)$ are polynomials

★ Exponential and Logarithmic Functions:

$$f(x) = a * b^x, b > 0, b \neq 1$$

$$y = \log_b x \leftrightarrow b^y = x$$

Review the Properties of logarithms from page 109
or see section 2.5 (page 3) of your class notes.

★ Limits:

Notation for limits from the left or right:

$$\lim_{x \rightarrow 5^-} f(x) \qquad \lim_{x \rightarrow 5^+} f(x)$$

To find a limit at a constant, say 5, plug in 5. If a zero occurs in the denominator, factor, reduce and try plugging in 5 again.

If from the left and right the graph is approaching infinity, then the limit is infinity.

If from the left and right the graph is approaching negative infinity, then the limit is negative infinity.

If from the left, the graph is approaching infinity, and from the right is approaching negative infinity, then the limit DNE (does not exist).

If from the left and right the graph is approaching two different y values, then the limit does not exist.

★ Continuity:

A function is continuous at $x = a$ if all three of the following conditions are met.

1. The limit of the function exists at $x = a$.
2. The function is defined when $x = a$.
3. The limit of the function as $x \rightarrow a$, is the same value as the function evaluated at a .

★ Difference Quotient:

$$\frac{f(x+h) - f(x)}{h}$$

To find the difference quotient for any function $f(x)$,

- a. find $f(x+h)$
- b. find $f(x+h) - f(x)$
- c. divide by h

★ Asymptotes:

Horizontal Asymptotes

Compare the degrees of the numerator and the denominator

- a. if the same, then there is a horiz asymptote at $y = \frac{a}{b}$,
the quotient of the leading coefficients

- b. if different, determine which has the larger degree
 1. if the numerator is of larger degree, there is NO horizontal asymptote
 2. if the denominator is of larger degree, then $y = 0$ is a horizontal asymptote

Holes in the Graph

If given a rational function, factor and see if any of the factors cancel. Set any factor that cancels equal to zero, and you have the location of a hole in the graph.

Vertical Asymptote: After canceling and reducing, and remaining factors can be set equal to

zero and solved, and you have the line, $x = a$, which is a vertical asymptote. There may be more than one vertical asymptote.

★ **Limits at Infinity:** finding horizontal asymptotes

★ **End Line Behavior:** answering the questions:

- a. As $x \rightarrow \infty$ what is happening to y ? or
- b. As $x \rightarrow -\infty$ what is happening to y ?

Find the degree of the polynomial.

- a. If even, both ends of the line head in the same direction as the sign of the leading coefficient. Plus = up and Negative = down
- b. If odd, both ends of the line head in opposite directions, one to infinity, and the other to negative infinity. To determine the direction, look at the sign of the leading coefficient.