

## Final Exam Review

MATH 142-Drost

1. Find the domain of the function  $f(x) = \frac{\sqrt{20-4x}}{\log(x+10)}$ .

$$20-4x \geq 0$$

$$-4x \geq -20$$

$$x \leq 5$$

$$x+10 > 0$$

$$x > -10,$$

$$\log(x+10) \neq 0$$

$$10^0 \neq x+10$$

$$1 \neq x+10$$

$$-9 \neq x$$

$$\boxed{(-10, -9) \cup (-9, 5]}$$

2. Find the vertex of  $f(x) = 4x^2 - 24x + 11$

$$h = \frac{-b}{2a} = \frac{-(-24)}{2(4)} = 3$$

$$(h, k) = (3, -25)$$

$$k = f(3) = 4(3^2) - 24(3) + 11$$

$$= 36 - 72 + 11 = \boxed{-25}$$

$$3. \lim_{x \rightarrow 2} \frac{x^2 + 3x - 10}{x^2 - x - 2} = \lim_{x \rightarrow 2} \frac{(x+5)(x-2)}{(x-2)(x+1)} = \lim_{x \rightarrow 2} \frac{x+5}{x+1}$$

$$= \frac{2+5}{2+1} = \boxed{\frac{7}{3}}$$

$$4. \lim_{x \rightarrow \infty} \frac{4x^2 - x + 5}{1 - x - 2x^2} = \frac{4}{-2} = \boxed{-2}$$

5. Describe the end line behavior of  $f(x) = ax^3 - bx^2 + c$ , if  $a, b, c < 0$ .

Odd power with negative leading  
Coefficient

$$\boxed{\begin{array}{l} \text{as } x \rightarrow -\infty, y \rightarrow \infty \\ \text{as } x \rightarrow \infty, y \rightarrow -\infty \end{array}}$$

6. Given  $\log_* 2 = a$ ,  $\log_* 3 = b$ , and  $\log_* 5 = c$ . Evaluate:  $\log_* 600$ .

$$\begin{aligned}\log_* (2^3 \cdot 3 \cdot 5^2) &= \log_* 2^3 + \log_* 3 + \log_* 5^2 \\ &= 3 \log_* 2 + \log_* 3 + 2 \cdot \log_* 5 \\ &= \boxed{3a + b + 2c}\end{aligned}$$

7. Solve:  $\log_2 \{ \log_3 [\log(x^2 + 90x)] \} = 0$

$$\begin{aligned}2^0 &= \log_3 (\log(x^2 + 90x)) & 0 &= (x-10)(x+100) \\ 3^0 &= \log(x^2 + 90x) & \boxed{x=10, x=-100}\end{aligned}$$

$$1000 = x^2 + 90x$$

$$0 = x^2 + 90x - 1000$$

8. Find the derivative of  $y = -4e^{2x} + 6x^2 - 15x + 7 + \ln(x^2 + 1)$ .

$$y' = -4(2)e^{2x} + 12x - 15 + \frac{2x}{x^2+1}$$

$$\boxed{y' = -8e^{2x} + 12x - 15 + \frac{2x}{x^2+1}}$$

9. Find  $h'(5)$  when  $h(x) = f(x) \cdot g(x)$  and  $f(5) = a - b$ ,  $g(5) = 3a + b$ ,  $f'(5) = 2a + 4b$ , and  $g'(5) = 2b$ .

$$\begin{aligned}h(x) &= f(x) \cdot g(x) \\ h'(x) &= f(x) \cdot g'(x) + g(x) \cdot f'(x) \\ h'(5) &= f(5) \cdot g'(5) + g(5) \cdot f'(5) \\ h'(5) &= (a-b)(2b) + (3a+b)(2a+4b) \\ h'(5) &= 2ab - 2b^2 + 6a^2 + 12ab + 2ab + 4b^2 \\ \boxed{h'(5) &= 6a^2 + 16ab + 2b^2}\end{aligned}$$

10. Find any critical values for  $f(x) = \frac{e^x}{2x+1}$ .

$$f' = \frac{(2x+1)e^x - e^x(2)}{(2x+1)^2}$$

$$f' = \frac{e^x(2x+1-2)}{(2x+1)^2} = \frac{e^x(2x-1)}{(2x+1)^2}$$

$$e^x(2x-1) = 0 \quad \text{when}$$

$$e^x = 0 \quad \text{or} \quad 2x-1=0$$

$$\emptyset \quad \quad \quad \boxed{x = \frac{1}{2}}$$

11. Given  $p = 36 - 2x$  is the price-demand function for Bubbles Car Wash, and  $x$  is the number of cars washed daily. What price maximizes the revenue?

$$R = x \cdot p = x(36 - 2x) = 36x - 2x^2$$

$$R' = 36 - 4x = 0 \Rightarrow 36 = 4x \Rightarrow x = 9$$

$$R'' = -4 \text{ concave down when } x=9$$

$p = 36 - 2(9) = \$18$

12. Find where the function  $f(x) = 10x - x \cdot \ln x$  is increasing and/or decreasing.

$$f(x) = 10x - x \ln x$$

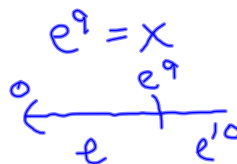
$$f(x) = x(10 - \ln x)$$

$$f' = x\left(-\frac{1}{x}\right) + (10 - \ln x)(1)$$

$$f' = -1 + 10 - \ln x$$

$$y' = 9 - \ln x = 0$$

$$9 = \ln x$$



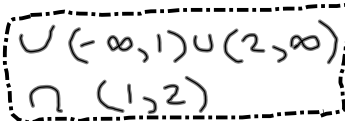
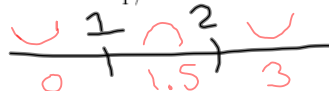
13. Given  $f(x) = x^4 - 6x^3 + 12x^2$ , find the intervals where  $f(x)$  is concave up/down.

$$f' = 4x^3 - 18x^2 + 24x$$

$$f'' = 12x^2 - 36x + 24 = 0$$

$$x^2 - 3x + 2 = 0$$

$$(x - 2)(x - 1) = 0$$



14. Given the price-demand function for each pint of double-chocolate chip ice cream is  $x = f(p) = 4p^2 - 12p$ . At a price of \$2.50/pint, is the price elastic, inelastic, or of unit elasticity?

$$f(p) = 4p^2 - 12p$$

$$f'(p) = 8p - 12$$

$$E(p) = \frac{-p \cdot f'(p)}{f(p)}$$

$$E(p) = \frac{-p(8p - 12)}{4p^2 - 12p}$$

$$E(p) = \frac{12p - 8p^2}{4p^2 - 12p}$$

$$E(p) = \frac{4p(3 - 2p)}{4p(p - 3)}$$

$$E(p) = \frac{3 - 2p}{p - 3}$$

$$E(2.5) = \frac{3 - 5}{2.5 - 3} = \frac{-2}{-1/2}$$

$$E(2.5) = 4$$

Elastic

15. If \$2500 is deposited at  $4\frac{1}{2}\%$  annual interest compounded continuously, how long before the balance is \$6000?

$$A = Pe^{rt}$$

$$P = 2500$$

$$r = .045$$

$$t = \boxed{\phantom{000}}$$

$$A = 6000$$

$$6000 = 2500 e^{.045t}$$

$$\frac{6000}{2500} = e^{.045t}$$

$$2.4 = e^{.045t}$$

$$\ln 2.4 = .045t$$

$$\frac{\ln 2.4}{.045} = t$$

$$t = 19.454860\dots$$

$t \approx 19.5$  yrs

16. Find any horizontal, vertical or oblique asymptotes that exist for each of the following functions:

a.  $f(x) = \frac{3x^2}{x^2 - 9}$

HA:  $y = 3$

VA:  $x = 3, x = -3$

OA: none

b.  $f(x) = \frac{5x^2}{x+5}$

HA: none

VA:  $x = -5$

OA:  $y = 5x - 25$

$$\begin{array}{r} 5x - 25 \\ x + 5 \overline{) 5x^2} \\ \underline{-5x^2 - 25x} \end{array}$$

c.  $f(x) = \frac{5x^2}{3x^2 + x}$

HA:  $y = \frac{5}{3}$

$f(x) = \frac{5x \cdot x}{x(3x+1)}$

VA:  $x = -\frac{1}{3}$

OA: none

$$\begin{array}{r} -25x + 0 \\ + -25x - 125 \\ \hline 125 \end{array}$$

17. Find each of the following indefinite integrals:

a.  $\int 5 dt = 5t + C$

b.  $\int (4x^2 + \sqrt{x} + 9) dx = \int (4x^2 + x^{1/2} + 9) dx =$

$$\frac{4}{3}x^3 + \frac{2}{3}x^{3/2} + 9x + C$$

c.  $\int \frac{4x-1}{2x^2-x} dx = \int \frac{\text{mess}'}{\text{mess}} dx = \ln|\text{mess}| + C$

$$= \ln|2x^2 - x| + C$$

d.  $\int \frac{4x-1}{(2x^2-x)^6} dx = \int (2x^2-x)^{-6} (4x-1) dx =$

$$\int u^{-6} du = \frac{1}{-5} u^{-5} + C =$$

$$-\frac{1}{5} (2x^2 - x)^{-5} + C$$

e.  $\int \frac{1}{x \cdot \ln x} dx =$

$$\int \frac{1}{\ln x} \cdot \frac{1}{x} dx =$$

$$\int \frac{1}{u} \cdot du = \ln|u| + C$$

$$= \ln|\ln|x|| + C$$

$$\begin{aligned}
 \text{f. } \int x \cdot e^{x^2+1} dx &= \int e^{x^2+1} (x) dx && \text{let } u = x^2 + 1 \\
 &= \int e^u \left(\frac{1}{2}\right) du && du = 2x dx \\
 &= \frac{1}{2} e^u + C = \frac{1}{2} e^{x^2+1} + C && \frac{1}{2} du = x dx
 \end{aligned}$$

$$\begin{aligned}
 \text{g. } \int \frac{x+2}{x+8} dx &= \int \frac{(u-8)+2}{u} du = \int \frac{u-6}{u} du = \int \left(1 - \frac{6}{u}\right) du \\
 u = x+8 & && \\
 du = dx & && \\
 u-8 = x & && \\
 & && = u - 6 \ln|u| + C = \\
 & && \boxed{x+8 - 6 \ln|x+8| + C}
 \end{aligned}$$

18. Find  $R_4$ , the approximate area under the function  $f(x) = x^3 + 6x^2 - 18x$  on the interval  $[-8, 0]$ .

$$\begin{aligned}
 \Delta x &= \frac{0 - (-8)}{4} = 2 && A \approx 2 \left( f_0 + f_{-2} + f_{-4} + f_{-6} \right) \\
 x &| -8 | -6 | -4 | -2 | 0 && A \approx 2(0 + 52 + 104 + 108) \\
 y &| 16 | 108 | 104 | 52 | 0 && \boxed{A \approx 528 \text{ units}^2}
 \end{aligned}$$

19. Find the area of the 2<sup>nd</sup> rectangle from the right of the previous problem.

$$\begin{aligned}
 A &= w \cdot h \\
 A &= 2(f_{-2}) = 2(52) = \boxed{104 \text{ units}^2}
 \end{aligned}$$

$$\begin{aligned}
 \text{20. Find the exact value of } \int_1^3 (9x^2 + \pi^2) dx &= \left( \frac{9x^3}{3} + \pi^2 x \right) \Big|_1^3 \\
 &= \left( 3x^3 + \pi^2 x \right) \Big|_1^3 \\
 &= (81 + 3\pi^2) - (3 + \pi^2) \\
 &= \boxed{78 + 2\pi^2}
 \end{aligned}$$

21. The value of our house can be modeled by the function  $V(t) = 174 + 5.9 \ln(t + 1)$ , with  $t$  representing the number of years since it was built in 1990, and  $V(t)$  is measured in thousands of dollars.

a. What does this model predict the value will be in 2010?  $\Rightarrow t = 20$

$$V(20) = 174 + 5.9 \ln(20+1)$$

$$= 191.9626824$$

$\$191,962.68$

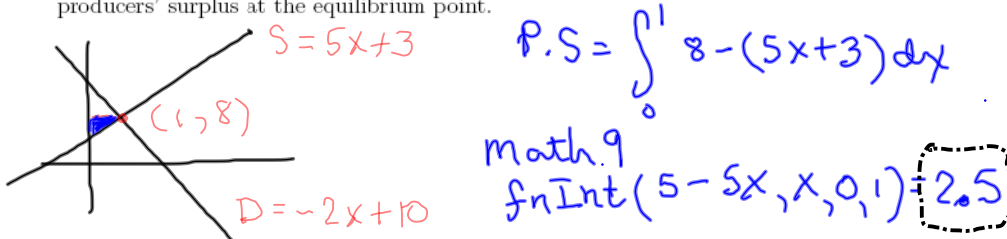
b. What was the average value of the house from 1990 to 2005?

$$\text{Avg Value} = \frac{1}{b-a} \int_a^b V(t) dt$$

$\$185,548.83$

$$= \frac{1}{15-0} \int_0^{15} (174 + 5.9 \ln(t+1)) dt$$

22. Given the demand function is  $p = -2x + 10$  and the supply equation is  $p = 5x + 3$ . Find the producers' surplus at the equilibrium point.



23. Find the maximum and minimum values, if they exist, for  $f(x) = 3x^2 - 1x - 2$ .

no Max

Min value  
 $-\frac{25}{12}$

$$f' = 6x - 1 = 0$$

$$x = \frac{1}{6}$$

$$f\left(\frac{1}{6}\right) = 3\left(\frac{1}{36}\right) - \frac{1}{6} - 2 = -\frac{25}{12}$$

24. Ellie's Electric Toys has fixed costs of \$2935/month and  $MC = 30x + 500 - e^{0.2x}$ . What is the cost function?

$$MC = C' = 30x + 500 - e^{0.2x}$$

$$C = \int (30x + 500 - e^{0.2x}) dx$$

$$C = 15x^2 + 500x - 5e^{0.2x} + K$$

$$C(0) = 0 + 0 - 5 + K = 2935$$

$$K = 2940$$

$C(x) = 15x^2 + 500x - 5e^{\frac{1}{5}x} + 2940$

25. Find  $f(2, -1)$  if  $f(x, y) = 2x - 3x^2y + 4y^2$ .

$$\begin{aligned} f(2, -1) &= 2(2) - 3(2)^2(-1) + 4(-1)^2 \\ &= 4 + 12 + 4 \\ &= 20 \end{aligned}$$

26. Find the first order partial derivatives of  $f(x, y) = 6x^2y + 2x - 3y^3$ .

$$f_x = 12xy + 2 \quad f_y = 6x^2 - 9y^2$$

27. Find the critical values of  $f(x, y) = 20x - 2x^2 + xy - y^2 + 2y$ .

$$\begin{aligned} f_x &= 20 - 4x + y = 0 & f_y &= x - 2y + 2 = 0 \\ y &= 4x - 20 & x - 2(4x - 20) + 2 &= 0 \\ y &= 24 - 20 & x - 8x + 40 + 2 &= 0 \\ y &= 4 & -7x &= -42 \\ & & x &= 6 \end{aligned}$$

$(6, 4)$

28. Is the critical point in the previous problem a relative minimum, relative maximum, or a saddle point?

$$\begin{aligned} f_{xx} &= -4 & f_{yx} &= 1 \\ f_{xy} &= 1 & f_{yy} &= -2 \end{aligned}$$

$D = -4(-2) - 1^2 = 8 - 1 > 0$

$f_{xx} < 0$  concave down

$\text{max}(6, 4)$

29. Find  $f_{yx}$  when  $f(x, y) = x^2y - 8x + 4xy + y^2 + 5y$

$$f_x = 2xy - 8 + 4y$$

$$f_{xy} = 2x + 4$$

30. Find the dimensions of a fish tank with minimum surface area, if the width must be 3 times the length, and the tank must hold 2700 cu in of water when full. Round your dimensions to two decimal places, if necessary.



$$w = 3l$$

$$V = 2700 = 3l \cdot l \cdot h \quad S_A = wl + 2lh + 2wh$$

$$\frac{900}{l^2} = h$$

$$S_A = 3l \cdot l + 2l \cdot \frac{900}{l^2} + 2 \cdot 3l \cdot \frac{900}{l^2}$$

$$S_A = 3l^2 + 1800l^{-1} + 5400l^{-1}$$

$$S_A = 3l^2 + 7200l^{-1}$$

$$S_A' = 6l - 7200l^{-2} = 0$$

$$6l = 7200l^{-2}$$

$$6l = \frac{7200}{l^2}$$

$$6l^3 = 7200$$

$$l^3 = 1200$$

$$l = \sqrt[3]{1200} = 10.63''$$

$$w = 3l = 31.88''$$

$$h = \frac{900}{l^2} = 7.97''$$

31. Given the production function for Allied Chemical of  $f(L, K) = 25L^{0.2} \cdot K^{0.8}$ , where  $L$  represent the investment in labor costs, and  $K$  represents the investment in capital.

a. Find the marginal productivity of labor at the current level of 50 units of labor and 30 units of capital.

$$f_L = 25(0.2) L^{-0.8} K^{0.8}$$

$$f_L = 5 L^{-0.8} K^{0.8} \quad f_L(50, 30) = 3.3227$$

b. Find the marginal productivity of capital at the current level of 50 units of labor and 30 units of capital.

$$f_K = 25(0.8) L^{0.2} K^{-0.2}$$

$$f_K = 20 L^{0.2} K^{-0.2} \quad f_K(50, 30) = 22.1513$$

c. At the current production levels of 50 units of labor and 30 units of capital, what will cause the greatest increase in the production level, an increase in units of labor or capital?

An increase of 1 unit in capital results in an increase of 22.15 units.

32. True or False:

a. The derivative of a constant is ALWAYS zero. True

b. The graph of a function ALWAYS has exactly one  $y$ -intercept. False

c. The graph of  $y = e^{x-a}$  has a vertical asymptote of  $x = -a$ . False

d.  $0.9 - 1$  True

e.  $2 * 5 - 6 = 0$  is an equation. True