

Review #Final Exam

MATH 142-Drost

- _____1. Find the domain of the function $f(x) = \begin{cases} \frac{\sqrt{x-1}}{x-2} & \text{if } x < 2 \\ \sqrt{5-x} & \text{if } x > 3 \end{cases}$
- _____2. Suppose 450 items are sold per day at a price of \$53 per item and that 650 items are sold per day at a price of \$45 per item. Find the demand function for p , assuming the demand curve to be a straight line.
- _____3. How much money would have to be invested at $6\frac{3}{4}\%$ compounded weekly to get back \$5000 at the end of 8 years? (Express answer rounded to the nearest cent.)
- _____4. Solve $3 \log(2x + 5) + 6 = 0$
- _____5. The price-demand function $p = \sqrt[3]{32 - x}$ models the price of Gary's Gadgets. Find the elasticity of demand when the price is \$1.50. Should the price remain the same, or be raised or lowered in order to increase revenue?
- _____6. Complete the square and put in standard form: $y = -2x^2 + 12x + 15$
- _____7. If $p = 40 - 2.5x$, find the marginal revenue at $x = 6$
- _____8. Estimate the cost of the 100th item if $C(x) = x^2 - 50x + 275$
- _____9. Given $f(x) = \begin{cases} e^{2 \ln a} & \text{if } x < 7 \\ x - 3 & \text{if } x \geq 7 \end{cases}$, for what value(s) of a is $f(x)$ continuous?
- _____10. $\lim_{x \rightarrow 2} \frac{x^2 - x - 2}{x^2 + x - 6}$

_____ 11. Given $f(x) = x^3 \cdot e^{4x}$, find $f'(x)$

_____ 12. Find the derivative of $f(x) = \sqrt{x^2 - 4}$

_____ 13. If \$12,000 is invested at $8\frac{1}{2}\%$ compounded continuously, how much is in the account in 8 years, assuming no withdrawals?

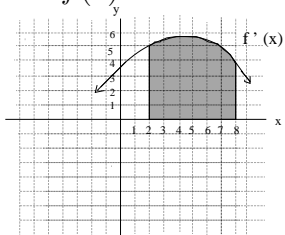
_____ 14. The function $f(x) = \frac{2x^2 + x - 1}{2x^2 + 23x - 12}$ has a vertical asymptote at:

_____ 15. Estimate: $\int_2^3 \sqrt[5]{4 + 8x^3} \cdot 4x \, dx$

_____ 16. Given marginal cost defined by $50e^x + 24x - 200$ and the company has fixed costs of \$150. Find the cost function.

_____ 17. If the $\int_a^b 12x^3 \, dx = 3840$ and $\int_a^4 12x^3 \, dx = 720$ find the sum of $a+b$, when $a > 0$ and $a < b$.

_____ 18. Given the graph of $y = f'(x)$. Suppose $f(8) = 72$ and the area of the shaded region is 56. Find $f(2)$.



_____ 19. Simplify: $\log_3 243 - e^{2 \ln 4} + \log 10$.

_____ 20. Given the supply equation $S(x) = 3x^2 + 50$ and the demand equation $D(x) = -2x^2 + 230$, find the producer's surplus.

_____21. Mr. Durham bought a giant screen TV for \$3800, new. It was expected to last 18 years with a scrap value of \$830. Assuming a linear depreciation, what is the set worth in five years?

_____22. Given $y = \frac{1}{3}x^2 - \frac{3}{4}x + 9$, locate the vertex.

_____23. Solve $3 \cdot 10^{2-5x} = 12$

_____24. Solve $\log(\log(5+x)) = 0$

_____25. An apartment has 80 units. When the rent is \$750 per month, all the units are rented. For each \$25 increase in rent, one apartment becomes vacant. Find the rent he should charge to maximize revenue.

_____26. The government of a small country is planning sweeping changes in the tax structure in order to provide a more equitable distribution of income. The Lorenz curve for the current income distribution is $f(x) = x^{2.3}$, and with the proposed changes, $g(x) = 0.4x + 0.6x^2$. Will the proposed changes provide a more equitable income distribution? Source: Calculus, by Barnett, Ziegler and Byleen, p. 433

_____27. $\lim_{x \rightarrow \infty} \frac{2e^x - x}{e^x + 3x}$

_____28. Given the following data, find the average rate of change on the interval $[3, 11]$.

x	3	5	7	9	11	15
y	9	17	21	29	31	63

_____29. If $C(x) = 2x^2 + 50$, find the number of items necessary to minimize the average cost per item.

_____30. Given $y = \frac{3x - e^{4x}}{\ln x}$, find $\frac{dy}{dx}$ and DO NOT SIMPLIFY

- _____ 31. Find $f''(x)$, the second derivative of $f(x) = x^2 \cdot \ln x$.
- _____ 32. Given: $f(x) = \frac{6 - 3x + 4x^2}{18 - 2x^2}$ Find the horizontal asymptote(s).
- _____ 33. Find the absolute minimum of $f(x) = x^3 - 3x^2 + 6$ on the interval $[0, 4]$.
- _____ 34. Estimate $\int_{-2}^1 e^{x^3} dx$ (to two decimal places) by finding the left-hand sums for $n = 3$.
- _____ 35. Find the EXACT value of $\int_{-2}^0 \sqrt{4 - x^2} dx$ by finding the area of the appropriate geometric region.
- _____ 36. Find the average value of $f(x) = 16 - x^2$ on the interval $[0, 4]$.
- _____ 37. Find the area enclosed by the curves: $y = x^2 - 6$ and $y = -x$.
- _____ 38. Given the supply equation $S(x) = p = 2x + 8$ and the demand equation $D(x) = p = 10 - 2x$. Compute the consumers' surplus.
- _____ 39. Evaluate $f(x, y) = \sqrt{8 - x - y^2}$ at $(-4, 2)$
- _____ 40. Find the domain:
- $f(x, y) = \frac{y}{x - 2y}$
 - $f(x, y) = \sqrt{8 - x + y}$
 - $f(x, y) = 3x^2 \sqrt{y}$
 - $f(x, y) = \sqrt[3]{16 + x^2 + y}$

- _____41. For $f(x, y) = e^{x^2+3y^2}$, find $f_y(2, 1)$.
- _____42. Find the surface area of a closed rectangular box whose volume is 1000 ft^3 as $S(x, y)$
- _____43. A company sells gadgets and widgets. The gadgets sell at $p = 120 - 2x - 3y$ and the widgets sell at $q = 200 - x - 5y$, where x = the number of gadgets sold and y = the number widgets sold. Find the revenue function, $R(x, y)$, and the value of $R(10, 20)$.
- _____44. Maximize the revenue in the previous problem.
- _____45. Find f_x and f_y given $f(x, y) = 3x + 4y^2 - 2xy$
- _____46. Find f_x and f_y given $f(x, y) = \sqrt{8 - x^2 - y^2}$
- _____47. Find f_x and f_y given $f(x, y) = x^4 \cdot e^{2xy}$
- _____48. Find f_x and f_y given $f(x, y) = \frac{x}{y^2 - 1}$
- _____49. Find the second partial derivatives of: $f(x, y) = 3x^2 + 4y^2 - 2x^2y^3$.
- _____50. Find the second partial derivatives of: $f(x, y) = e^{x-2y}$

_____ 51. The Bollinger Processing Plant is producing palm trees, x = the number of small trees, and y = the number of pairs of large trees. The small trees sell for \$25 each, and the large trees sell for \$100 per pair. The cost of producing these trees is given by the function $C(x, y) = 5x^2 - 10xy + 10y^2$. Find the number of each they should produce to maximize profit.

_____ 52. Find where the function $f(x) = x^2 - \ln x$ is decreasing.

_____ 53. Determine the interval over which $f(x) = x^3 - 6x^2 - 15x + 10$ is concave down.

_____ 54. A video store expects to sell 10,000 copies of **Chicago** during the coming year. It costs the company 10 cents to store a video for a year, and there is a \$125 shipping and handling fee for each order. If each video costs the store \$8, what size orders should they place to minimize inventory costs?

_____ 55. $\int (x - 2)e^{3x^2 - 12x} dx$

_____ 56. Find the absolute extrema for $f(x) = 12 + 4x + \frac{25}{x}$ on the interval $[2, 10]$

_____ 57. Find $f(x)$ given $f'(x) = 4e^{8x} + 2$ if $f(0) = 5$.

_____ 58. $\int 4^{x^2+1}(4x) dx$

_____ 59. Given $f(x) = \begin{cases} 6-x & \text{if } x < -10 \\ 2x^2 + 15x - 30 & \text{if } -10 \leq x < 0 \\ -10 + 5 \log_2\left(\frac{1}{16}\right) & \text{if } x = 0 \\ -(2^{5 \log_2(x+2)} - 2) & \text{if } x > 0 \end{cases}$

For what interval(s) is $f(x)$ continuous?

_____ 60. Given $f(x) = x^2 - 1$, $g(x) = \frac{x}{x-3}$, and $h(x) = \sqrt{7-x}$. Find $(f \circ g \circ h)(3)$

Answers:

1. $[1, 2) \cup (3, 5]$
2. $p = -.04x + 71$
3. \$2,914.76
4. $x = -2.495$
5. .3537, raised to increase revenue
6. $y = -2(x - 3)^2 + 33$
7. 10
8. \$148
9. $a = 2$
10. $\frac{3}{5}$
11. $f'(x) = x^2(e^{4x})(4x + 3)$
12. $f'(x) = \frac{x}{\sqrt{x^2 - 4}}$
13. \$23,686.53
14. $x = -12$
15. 26.6031
16. $C(x) = 50e^x + 12x^2 - 200x + 100$
17. 8
18. 16
19. -10
20. \$432
21. \$2,975.
22. $\left(\frac{9}{8}, \frac{549}{64}\right)$
23. $x = \frac{1}{5}(2 - \log 4)$
24. $x = 5$
25. \$1375.
- 26.
27. 2
28. 2.75
29. $x = 5$
30. $\frac{dy}{dx} = \frac{(\ln x)(3 - 4e^{4x}) - (3x - e^{4x})\left(\frac{1}{x}\right)}{(\ln x)^2}$
31. $f''(x) = 3 + \ln x^2$
32. $y = -2$
33. 2
- 34.
35. π
36. $10\frac{2}{3}$
37. $20.8\bar{3}$
38. .25
39. $2\sqrt{2}$
40. a) $y \neq \frac{x}{2}$
- b) $y \geq x - 8$
- c) $y \geq 0$
- d) \Re
- 41.
42. $S_A = 2000y^{-1} + 2000x^{-1} + 2xy$
43. $R(x, y) = 120x - 2x^2 - 4xy + 200y - 5y^2$
- $R(10, 20) = 2200$
44. $x = 16\frac{2}{3}, y = 13\frac{1}{3} \Rightarrow R = 2333.\bar{33}$
45. $f_x = 3 - 2y, f_y = 8y - 2x$
46. $f_x = \frac{-x}{\sqrt{8 - x^2 - y^2}}, f_y = \frac{-y}{\sqrt{8 - x^2 - y^2}}$
47. $f_x = 2x^3e^{2xy}(xy + 2), f_y = 2x^5e^{2xy}$
48. $f_x = \frac{1}{y^2 - 1}, f_y = \frac{-2xy}{(y^2 - 1)^2}$
49. $f_{xx} = 6 - 4y^3, f_{xy} = -12xy^2, f_{yx} = -12xy^2, f_{yy} = 8 - 12x^2y$
50. $f_{xx} = e^{x-2y}, f_{xy} = -2e^{x-2y}, f_{yx} = -2e^{x-2y}, f_{yy} = 4e^{x-2y}$
51. maximum profit with 15 small trees, and 25 large trees
52. $(0, \frac{1}{\sqrt{2}})$
53. $(-\infty, 2)$
54. 5,000 videos
55. $\frac{1}{6}e^{3x^2-12x} + C$
56. MAX = 54.5, MIN = 32
57. $f(x) = 0.5e^{8x} + 2x + 4.5$
58. $\frac{2 \cdot 4^{x^2+1}}{\ln 4} + C$
59. $(-\infty, -10), (-10, \infty)$
60. 3