Math 142 Lecture Notes
Section 1.1 – Linear Equations and Inequalities

Equation:

Definition:

A statement of equality.

Which of the following represent an equation?

1) $14 - 5 = 2$

2) $25 + 4 = 29$

3) $3x - 27 = 33$

4) $5x - 4y$

5) $2x - 5y \le 10$

Linear Equations, and Inequalities

Linear Equations:

$Ax + B = C, \quad A \neq 0$

To Solve an Equation: means to find all values of the variable which make the equation (or inequality) true.

Linear Inequalities:

To write an inequality replace the equal sign in an equation with the symbols,

$\le$, $\ge$, $<$, $>$.

Equality Properties

1. An equivalent equation results if the same quantity is added or subtracted from both sides of the equation.
2. An equivalent equation is formed if both sides of the equation are multiplied or divided by the same nonzero number.

Example 1 : Solving a linear equation

$4x - 2(x + 3) = 5(x - 2) + 9$

_ remove parenthesis

_ combine like terms

_ subtract $2x$ from both sides of the equation

_ add 1 to both sides of the equation

_ divide both sides by 3

☆Example 2 : Solving a linear equation

$$\frac{x-4}{3}+\frac{x}{5}=4$$

_ multiply through both sides by the LCD

_ remove parenthesis

_ combine like terms

_ add 20 to both sides of the equation

_ divide both sides by 8

Note: In both examples above, the last step to **CHECK** your solution is optional. There are some steps, such as squaring both sides of an equation, for which you **must** check to see if the answers really work.

☆Example 3 : Solving a Formula for a particular variable

Solve the equation $P = a \cdot b + b \cdot c \cdot d + c$ **for "b".**

_ isolate the variable (every term with **b** should be on one side of the equation)

_ factor out the variable **b**

_ divide both sides of the equation by the factor

Solve the equation $P = a \cdot b + b \cdot c \cdot d + c$ **for "c".**

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☆ Linear Inequalities

Definition of "less than" **:** $a < b$ if $a + c = b, \quad c > 0$

Examples of Inequalities:

$3 < 5$

$-8 < -2$

$0 > -2$

Geometric Interpretation of "less than" : $a < b$ means a is to the left of b on the number line.

Theorem 2: Inequality Properties

An equivalent inequality will result and the sense or direction **will remain the same** if
1. the same real number is added or subtracted from each side.
2. both sides are multiplied or divided by the same positive number.

An equivalent inequality will result and the sense or direction **will reverse** if
3. each side is multiplied or divided by the same negative number.

Solve: $2x - 8 < 16 - 6x$

Double Inequalities

1) definition: $a < x < b$ means $a < x$ **AND** $x < b$

2) notation :

inequality form	**OR**	interval form
$a < x < b$		(a, b)
$a \le x < b$		$[a, b)$
$a < x \le b$		$(a, b]$
$a \le x \le b$		$[a, b]$

3) Solve: $5(2x - 3) \le 4(x + 1) - 1$ Express your answer in both a) inequality form and b) interval form.

4) Solve and graph your solution: $-11 \le 3x - 5 < 7$

Procedure for Solving Word Problems.

1. Read the problem and pick a variable to represent the unknown.
2. Identify all other unknowns and label in terms of the variable previously chosen in step 1.
3. Write an equation (or inequality) to represent the problem stated.
4. Solve
5. Check

Example : Purchase

LaDanian buys a $60"$ plasma screen online for $\$5400$, from a company which charges a $\$125$ shipping and handling fee, in addition to a $7\frac{3}{4}\%$ sales tax. What is the total price of the plasma screen?

Break-Even Analysis

Definition of terms:

1. Revenue: revenue = how many • how much
2. Cost: C = total costs
3. When $R < C$, the company has a **loss**.
4. When $R = C$, the company **breaks even**.
5. When $R > C$, the company has a **profit**.

Costs

1. fixed costs: those costs that exist when **zero** items are produced.
2. variable costs: those costs that depend on the number of item produced (cost per item)
3. $C = f + v \bullet x$ Total costs = fixed cost + variable costs • number of items

Example: A book company produces children's books. One time fixed costs for *Little Home By The Sea*, *are* $\$12{,}838$ which includes fees to the author, the printer and for the building. Variable costs amount to $\$14.50$ per book. The books are then sold to bookstores around the country at $\$39.00$ each. How many books must be printed and sold to *break-even*?