Math 142 Lecture Notes
Section 1.3 – Linear Regression

Mathematical Modeling:

Definitions:

Using mathematics to solve real-world problems.

Three Steps:

1) Construct the mathematical model.
2) Solve the mathematical model.
3) Interpret the solution to the mathematical model.

Slope as a Rate of Change

When the variables x and y are related by the equation $y = mx + b$, then they have a *linear* relationship and the slope is $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{change\ in\ y}{change\ in\ x}$.

When looking at application problems this is called the *rate of change of* y with respect to x.

Examples of rate of change

1) miles per hour = $\dfrac{number\ of\ miles}{number\ of\ hours}$
2) revolutions per minute
3) price per pound
4) miles per gallon

When the relationship is not linear, the ratio is called the **average rate of change of** y **with respect to** x.

Example 1: Appropriate doses of medicine for both animals and humans are often based on body surface area, BSA. Since weight is much easier to determine than BSA, veterinarians use the weight of an animal to estimate BSA. The following linear equation expresses BSA for canines in terms of weight, $a = 16.12w + 375.6$, where a is the BSA in square inches, and w is the weight in pounds.

Source: Calculus for Business, Economics, Life Sciences and Social Sciences by Barnett, Ziegler, Byleen, 11th edition, pg 30

Questions:

1. What is the slope of the linear equation?
2. Interpret.
3. What is the effect of a 1 pound increase in weight?

Example 2: A 100-pound cargo of delicate electronic equipment is dropped from an altitude of 2,880 feet and lands 200 seconds later. Source: Calculus for Business, Economics, Life Sciences and Social Sciences by Barnett, Ziegler, Byleen, 11th edition, pg 30

Questions:

1. Find a linear model relating altitude a (measured in feet) and time in the air t (measured in seconds).

2. How fast is the cargo moving when it lands?

Linear Regression

Regression Analysis is the process of finding a function that fits the mathematical data.

1) also referred to as **curve fitting**

2) using a graphing calculator to find the best fit

Example 3: Diamond Prices
Prices for round-shaped diamonds taken from an online trader are given in Table 1 (shown below).
Source: Calculus for Business, Economics, Life Sciences and Social Sciences by Barnett, Ziegler, Byleen, 11th edition, pg 31

Weight (carats)	Price
0.5	$2,790
0.6	$3,191
0.7	$3,694
0.8	$4,154
0.9	$5,018
1	$5,898

A linear model for the data is given by $p = 6{,}140c - 480$ where p is the price of a diamond weighing c carats.

Questions:

1. Create a scatter plot of the data and graph the model.

2. Interpret the slope of the model.

3. Use the graph to estimate the price of a diamond weighing 0.85 carats.

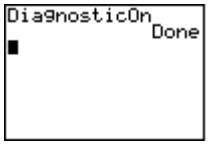
4. Use the graph to estimate the weight of a diamond whose price is $4,000.

☆Linear Regression on your Calculator: Using the TI-84 or TI-83 to find a linear regression

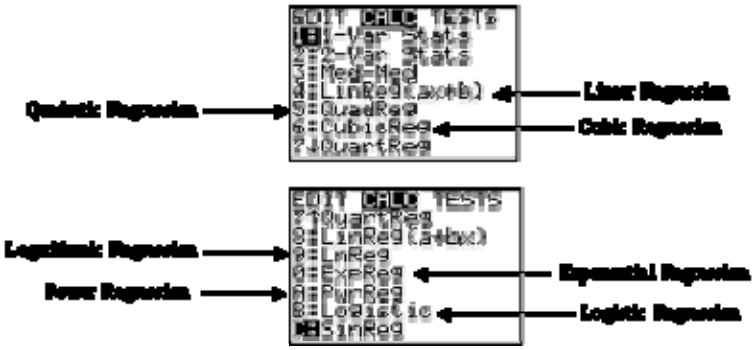
Source: **Jenn Whitfield's TI-83 Tutorial for Business Calculus**
http://www.math.tamu.edu/~jwhitfld/TI_83/TI83_tutorial.html#top

Regression (finding a best fitting model)

1. Activating the R^2
   - Press 2nd 0 to access the catalog.
   - Arrow down to DiagnosticOn and press ENTER. This will take you to the home screen.
   - Press ENTER and the calculator will respond with done.



2. Press Y= and clear all functions.
3. Follow the instructions for creating scatterplots to display the data. (See pg 4 of notes)
4. Press STAT and ▶ to access the CALC menu. Select the number that corresponds with the type of regression needed.



Once you have chosen the type of regression needed, your calculator will automatically go to the home screen.

5. Press VARS and ▶ to *Y-VARS*.
6. Select ***1:Function***. Then select ***1:Y1*** and press ENTER. The regression information, along with the R^2 value, will appear on the screen.
7. Press Y= and the regression equation appears in Y1.
8. Press GRAPH to see the data points and regression model displayed. You may need to adjust your window to see the graph.
9. To find other regression models for the same data, go back to step 5 and repeat the process.

CREATING A SCATTERPLOT

1. Clear all data from lists by pressing STAT and choosing *4:ClrList*. This will take you back to the home screen.
2. Now press 2nd 1 , 2nd 2. Your screen should like the following.

If it does then press ENTER and the calculator should respond with **Done**.

3. Press STAT then select ***1:Edit...***

4. Input the *x* values into L_1 and the *y* values into L_2.
5. Press 2nd Y= to access the scatterplot menu. Select ***1:Plot 1...*** Make the screen look like the folllowing

6. Press GRAPH and adjust your window so all data points can be seen on the screen.

Note: The instructions above are for all types of regression formulas, although in this lesson we are primarily looking at **linear regressions**, or straight lines.

Example 4: The table below shows the annual average price, median price and units sold for both single-family homes and condos/townhomes in Laguna Beach, California.

Source: http://orangecoastrealestate.com/annual/lb.html

Define t as number of years since 1990, and enter the data, as instructed above in your calculator in lines 1, 2, 3, and 4. First select **stat**, **edit**, then enter **data**. The first column will have to be redefined.

Year	Average Price	Median Price	Number Sold
1998	$ 764,547	$ 549,000	519
1999	$ 836,438	$ 618,250	483
2000	$1,043,451	$ 725,000	494
2001	$1,155,027	$ 802,000	380
2002	$1,129,204	$ 850,000	446
2003	$1,464,398	$1,050,000	451
2004	$1,698,760	$1,322,500	418
2005	$1,934,760	$1,597,000	336
2006	$2,521,450	$1,732,500	270

Questions:

1. Find a linear model that relates the average price and the year sold, defining t as the number of years since 1990 and the average price as P.
   a. What does the slope represent?

   b. When does this model predict the average price will be $3 million?

2. Find a linear model that relates the median price and the year sold, defining t as the number of years since 1990 and the median price as P.
   a. What is the increase/decrease predicted to be per year in the median price?

   b. What does this model predict the median price will be in 2008?

3. Find a linear model that relates the number sold and the year sold, defining t as the number of years since 1990 and N as the number sold that year.
   a. What does the slope represent?

   b. How many units does this linear regression predict will be sold in 2007?

☆**Best Fitting Line :** r, the correlation coefficient, is only one predictor of the best fitting line.

Last but not least, to find the best fitting line, you must take into consideration two factors.

1. Find the highest r value if only increasing or decreasing; (use the r^2 value when the curve is both increasing and decreasing).
   Highest means closest to 1 or -1.

   AND

2. The model selected must make sense.