

Math 142 Lecture Notes Section 2.1 – Functions

★ Equations in Two Variables

- 1) $Ax + By = C$ is a linear equation, and to graph this equation, one needs to plot only two points (three to check your work).
- 2) Graphing higher order equations

★ Graphing: Point by Point

- 1) Sketch the graph of : $y = 5 - 4x^2$

X	Y

- 2) Sketch the graph of : $y = -x^3 - 3x$

X	Y
-1	4
0	0
1	-4

- a) Plotting the three points listed above would give us the graph of a straight line.
- b) Is that accurate?
- c) Suggestions:
 - 1) check additional points
 - 2) How many additional points ?
 - 3) Check the graph with your graphing calculator.
- d) point-by-point plotting has it's limitations

Function: a rule that pairs elements between two sets, such that every element in the first set is paired with one element in the second set AND it is paired with **only one item** in the second set.

- 3) domain:
- 4) range:

Example 1: Label each of the following as a relation (correspondence) **OR** a function.

- 1) pair each student in the room with their phone number
- 2) pair each student in the room with their city of birth
- 3) pair each natural number with its cube
- 4) pair each student with the type of car they own

Functions Defined by Equations

- 1) Revenue is determined by multiplying how many “things” were sold times the price.
Let R = revenue, and x = the number of items sold $R = x \cdot p$
If the price is given as \$8.00, then the revenue equation becomes: $R = 8x$

Input:

Output:

Functions Defined By Equations:

In an equation with two variables, if each input value has one and only one output value, the equation represents a function.

Example 2: Label each of the following as a relation (or correspondence) **OR** a function.

- 1) $5x - 3y = 15$ where x is a real number
- 2) $y = x^2 - 6$ where x is a real number
- 3) $x^3 - y = 27$ where x is a real number
- 4) $x - y^2 = 0$ where x is a real number
- 5) $y = 7$ where x is a real number
- 6) $x = 3$ where y is a real number

★ Vertical-Line Test for a Function

If a vertical line passes through more than one point, the graph is NOT a function.

If a vertical line passes through at most one point, the graph is a function.

Agreement: Let's agree to the following guidelines:

- 1) If not stated, the domain is the set of all real numbers that produce real values.
- 2) In many applied problems, the domain is determined by the constraints of the word problem.

★ Finding the Domain

The domain of a function is all Reals, **unless**

- a) the function is a fraction, then exclude any value which would make the denominator zero.
- b) the function is an even-indexed radical, then the quantity under the radical sign must be non-negative.
- c) the function is a logarithm, then the quantity you are taking the log of must be positive.
- d) the function represents a word problem, in which case the domain is restricted to those values which make sense.

1) $y = \sqrt{3x - 12}$

2) $y = \frac{\sqrt{2 - x}}{x + 5}$

- 3) Let x = the number of fence posts, and y = the perimeter of the corral

★ Function Notation

1) $f(x)$ $(x, f(x))$

2) $f(x) = 4x - 10$, find $f(-2)$

3) $g(x) = 10 - 3x$, find $g(2)$

4) $h(x) = \sqrt{x - 2}$, find $h(1)$

5) Using the functions as defined above, find: $f(3) + 2g(-2) - 6h(11)$

★ Difference Quotient: If $f(x) = x^2 + 3x$ find each of the following:

1) $f(x + h)$

2) $f(x + h) - f(x)$

3) $\frac{f(x + h) - f(x)}{h}$

★ Applications

Important Terms

- a) costs and revenues
- b) break even, loss, profit
- c) fixed costs and variable costs

Important Formulas
Cost Function

Price-Demand Function

Revenue Function

Profit Function



Price-Demand

Source: *Calculus*, Barnett, Ziegler, Byleen, 11th Edition, pg.56

X (in millions)	P (in dollars)
2	87
5	68
8	53
12	37

A manufacturer of a popular digital camera wholesales the camera to retail outlets throughout the U.S. Through regression analysis the price-demand equation was determined to be: $p(x) = 94.8 - 5x$ for $1 \leq x \leq 15$

Cost Data

X (millions)	C(x) (millions \$)
1	175
5	260
8	305
12	395

$$C(x) = 156 + 19.7x$$

Revenue

X (millions)	R(x) (millions \$)
1	90
3	239
6	389
9	448
12	418
15	297

$$R(x) = x \cdot p(x) = x(94.8 - 5x)$$

Profit

X (millions)	P(x) (millions \$)
1	-86
3	
6	
9	
12	
15	