

## Math 142 Lecture Notes

### Section 2.2 – Elementary Functions: Graphs and Transformations

#### ★ Basic Elementary functions

Know the basic shapes of each of these functions

- 1) Identity function  $f(x) = x$
- 2) Square function  $g(x) = x^2$
- 3) Cube function  $h(x) = x^3$
- 4) Square root function  $m(x) = \sqrt{x}$
- 5) Cube root function  $n(x) = \sqrt[3]{x}$
- 6) Absolute value function  $p(x) = |x|$

**definition:** 
$$|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

**ABS(x)** is used on computers and calculators to represent the absolute value of  $x$ .

**Note:**  $\sqrt{x^2} = |x|$

#### ★ Vertical and Horizontal Shifts

**Vertical Shifts:**

- 1) Describe the graph of  $y = x^2 - 3$ , compared to the graph of its parent function.
- 2) Describe the graph of  $y = \sqrt{x} + 2$ , compared to the graph of its parent function.
- 3) Describe the graph of  $y = |x| - 5$ , compared to the graph of its parent function.

**Horizontal Shifts:**

- 1) Describe the graph of  $y = (x - 4)^3$ , compared to the graph of its parent function.
- 2) Describe the graph of  $y = \sqrt[3]{x + 1}$ , compared to the graph of its parent function.

3) Describe the graph of  $f(x) = (x+3)^2 - 4$ , compared to the graph of its parent function.

4) Describe the graph of  $g(x) = |x-4| + 2$ , compared to the graph of its parent function.

★ **Reflections, Expansions, and Contractions:** also called reflections, stretches, and shrinks!

1)  $y = -|x|$

2)  $y = -|x+2|$

3)  $y = -|x+2| + 3$

4)  $y = 3|x|$

5)  $y = \frac{1}{2}|x+2|$

★ **Piecewise-Defined Functions**

1) 
$$f(x) = \begin{cases} x^2 - 3, & x > 1 \\ -|x+4|, & x \leq 1 \end{cases}$$

2) Write a piecewise-defined function to represent the following problem.  
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Be sure to define your variables!

★ **Order Matters !**

Try the following transformations in the order listed. Are the end results identical?

1. Graph  $y = x^2$ , then shift left one unit, down two units, then reflect about the x-axis.

2. Graph  $y = x^2$ , reflect about the x-axis, shift down two, and one unit left.

3. *If order matters, what is the correct order ?*