Math 142 Lecture Notes
Section 2.5 – Logarithmic Functions

☆**One-to-one Functions**:

A function is said to be **one-to-one** if each range value corresponds to exactly one domain value.

A) $f(x)=\frac{x}{3}$ and $g(x)=\frac{|x|}{3}$ *Which function is one-to-one?*

B) $f(x)=x^3$ and $g(x)=x^2$ *Which function is one-to-one?*

C) $f(x)=|x+2|$ and $g(x)=\sqrt{x-2}$ *Which function is one-to-one?*

One-to-one function: If the graph of a function passes a ***horizontal line test***, any horizontal line passes through one and only one point, then the function is ***one-to-one.***.

Important points:

1) Any function that is either increasing or decreasing over the entire domain is ***one-to-one.***

2) If a continuous function increases for some domain values and decreases for others, it ***cannot*** be one-to-one.

Definition: Inverse of a Function

1. If *f is one-to-one*, then the inverse of *f* is the function formed by interchanging the independent and dependent variables for f.

2. If (a, b) is a point on the graph of *f* , then (b, a) is on the graph of the inverse of *f*.

3. If *f* is not one-to-one, it does not have an inverse.

☆**Logarithmic Functions.**

The inverse of an exponential function is called a ***logarithmic function***.

Compare the graphs of each of the following:

1. $y=2^x$ and $y=\log_2 x$

Compare the domain and ranges of each of the following:

2. $y=e^x$ and $y=\ln x$

Changing from logarithmic to exponential form:

$\log_b x = a \Leftrightarrow b^a = x$ where $b > 0$ and $b \neq 1$.

Change each of the following to exponential form:

1. $2 = \log_b 16$

2. $\ln x = 3$

3. $y = \log x^2$

Change each of the following to logarithmic form:

1. $5^3 = 125$

2. $8 = \sqrt{64}$

3. $\frac{1}{8} = 2^{-3}$

Domain and Range of Logarithmic Functions.

1. $y = \ln(x-5)$
   Domain: ______________ Range: ______________

2. $y = 5\log_2(x+4)$
   Domain: ______________ Range: ______________

3. $y = 3 - \log(16-2x)$
   Domain: ______________ Range: ______________

Logarithmic Equations.

Solving equations of the form $y = \log_b x$ **Find y, b or x as indicated.**

1. Find y: $y = \log_3 81$

2. Find b: $\log_b 11 = \frac{1}{2}$

3. Find x: $\log_3 x = 3$

Properties of Logarithmic Functions : If b, M, and N are positive real numbers, $b \neq 1$, and p and x are real numbers, then

1. $\log_b 1 = 0$
2. $\log_b b = 1$
3. $\log_b b^x = x$
4. $b^{\log_b x} = x, x > 0$
5. $\log_b MN = \log_b M + \log_b N$
6. $\log_b \frac{M}{N} = \log_b M - \log_b N$
7. $\log_b M^p = p \log_b M$
8. $\log_b M = \log_b N$ if and only if $M = N$

Using Logarithmic Properties: Rewrite each of the following expressions without a product or quotient.

Example 1: $\log_b \frac{xyz}{w}$

Example 2: $\log_b (xy)^{3/4}$

Example 3: $e^{x \ln 2}$

Example 4: $\frac{\log_e x}{\log_e b}$

Solving Logarithmic Equations:

Solve for x: $\frac{3}{2}\log_b 4 - \frac{2}{3}\log_b 8 + \log_b 2 = \log_b x$

Solve for x: $\log_3 x + \log_3 (x-3) = \log_3 10$

☆Logarithmic Notation:

Common logarithm: base 10 $\log x = \log_{10} x$

Natural logarithm: base *e* $\ln x = \log_e x$

☆Approximating solutions to Exponential Equations:

approximate your solutions to 4 decimal places.

Solve: $10^x = 5$

Solve: $e^x = 8$

Solve: $\log_4 25 = x$

Solve: How long (rounded to the nearest year) will it take for an investment to double if it is invested at 18% compounded semi-annually?