

Math142 Lecture Notes

Section 3.2 - Continuity

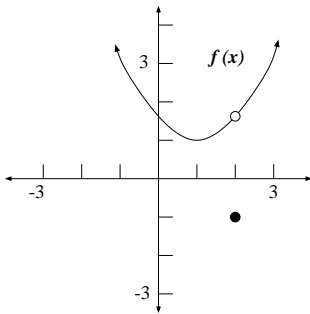
Definition of Continuity

A function, f , is said to be continuous at the point $x = a$ if all the following are true.

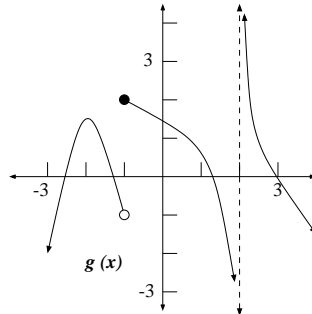
1. $\lim_{x \rightarrow a} f(x)$ exists.
2. $f(a)$ exists.
3. $\lim_{x \rightarrow a} f(x) = f(a)$.

(A continuous function has no holes, gaps, or breaks in its graph.)

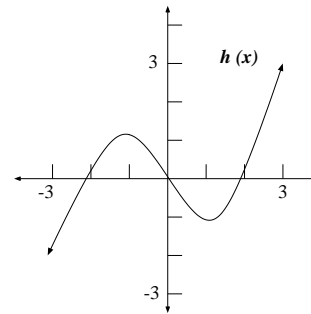
Example 1: Find all points of discontinuity in the graphs below. If f is not continuous at a point, state which of the three conditions given in the definition of continuity is violated.



Points of Discontinuity:



Points of Discontinuity:



Points of Discontinuity:

Example 2: Determine the continuity of $f(x) = \frac{x - 4}{x^2 - 16}$ at the indicated points.

- (a) at $x = 4$
- (b) at $x = -4$
- (c) at $x = 0$

Example 3: Determine the continuity of $g(x) = \begin{cases} x - 3 & x \leq 0 \\ x^2 - 3 & 0 < x \leq 1 \\ x & x > 1 \end{cases}$ at

(a) $x = 0$

(b) $x = 1$

Example 4: Find the value of k that will make $g(x) = \begin{cases} -x + 2k & x \geq 4 \\ 2x & x < 4 \end{cases}$ continuous on $(-\infty, \infty)$.

Functions that are Continuous on $(-\infty, \infty)$

- Polynomial
- Exponential

Functions that are Continuous on their domain

- Natural Logarithm
- Radical (Rational Exponent)
- Rational (discontinuous at values of x that make the denominator 0).

Example 5: Determine the intervals where the following functions are continuous.

(a) $f(x) = e^{0.5x}$

(b) $f(x) = \sqrt[3]{x^2 - 9}$

(c) $f(x) = \sqrt{5x - 15}$

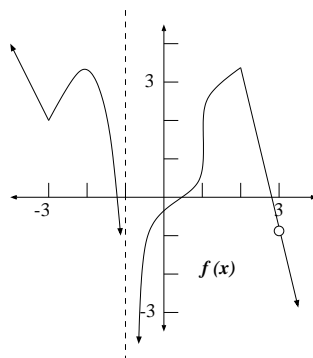
(d) $f(x) = \frac{4x - 5}{(4x - 5)(x + 2)}$

(e) $g(x) = 4x^2 - 4x + 1$

Continuity Properties of Some Specific Functions

- A constant function $f(x) = k$, where k is some constant, is continuous for all x .
- $f(x) = x^n$ is continuous for all x , where n is a positive integer.
- A polynomial function is continuous for all x .
- A rational function is continuous for all x **except** those values that make a denominator 0.
- For all n which are odd positive integers greater than 1, $\sqrt[n]{f(x)}$ is continuous wherever $f(x)$ is continuous.
- For n an **even** positive integer, $\sqrt[n]{f(x)}$ is continuous wherever $f(x)$ is continuous and nonnegative.

Example 6: Determine the intervals over which the following function is continuous:



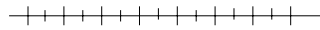
Example 7: Determine where $f(x) = |x + 2| - 3$ is continuous.

Sign Properties on an Interval

- If $f(x) \neq 0$ over the interval (a, b) , then either $f(x) > 0$ for all x in (a, b) or $f(x) < 0$ for all x in (a, b) .

Example 8: Solve each of the following inequalities using a sign chart.

a) $\frac{x - 8}{2x + 4} > 0$



b) $\frac{x^2 - 25}{x^2 - 9} \leq 0$

