Math 142 Lecture Notes
Section 3.4 – The Derivative

☆ Rate of Change:

Examples:

1) Traveling 65 mph, means a change in distance (65 miles) with respect to a given time period (1 hour).

2) A cold front is expected tonight and is measured in rate of change of degrees over a given time period. (-5^{o} per hour).

3) The slope of a line is the rate of change of the y-values, related to the changes in the x-values. $\frac{\Delta y}{\Delta x} = \frac{change\ in\ y}{change\ in\ x}$

4) The profit (in dollars) from the sale of t-shirts at Paula's Palace is given by $P(x) = 50x - 0.05x^2$.

   a. What is the profit when 100 t-shirts are made and sold?

   b. What is the profit when 500 t-shirts are made and sold?

   c. What is the average change in profit if they increased production from 100 t-shirts to 500 t-shirts, made and sold?

☆ Average Rate of Change:

For $y = f(x)$, the **average rate of change from x = a to x = a + h** is

$$\frac{f(a+h) - f(a)}{(a+h) - a} = \frac{f(a+h) - f(a)}{h} \qquad h \neq 0$$

This is called the difference quotient.

Example 1 : A water balloon dropped from the top of the apartment complex will fall a distance of y feet in x seconds, as given by the formula (from physics):

$$y = f(x) = 16x^2$$

- **Find the average velocity from x = 3 seconds to x = 4 seconds.**

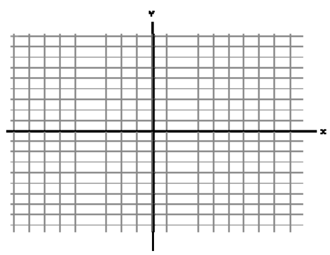
- **Find the average velocity from x = 3 seconds to x = (3 + h) seconds.**

- **Find the limit of the expression as h $\to$ 0.**

Slope of a Graph: The slope of the graph of y = f (x) at the point (a, f(a)) is given by: $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$ **provided that the limit exists**.

Note: The slope of the graph is also the slope of the tangent line at the point.

Example 2 : Given $f(x)=x^2-6x+12$



1. Find the slope of the secant line for a=3, and h=1.

2. Find the slope of the secant line for a=3 and h any nonzero number.

3. Find the limit of the expression in part 2, above.

4. Interpret your answer.

The Derivative

For y = f (x) , we define the derivative of *f* at x, written *f'(x)* , to be

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$ **if the limit exists.**

Interpretations of the Derivative.

1. The slope of the tangent line.
2. The instantaneous rate of change.
3. Velocity.

Examples:

A. Find f '(x) , using the limit definition of the derivative for the function, $f(x) = 3x - x^2$

B. Find the slope of the graph of f (x) at x = 0, x = 3, x = -2.

C. Find f ' (x), using the limit definition of the derivative for the function, $f(x) = \sqrt{x+4}$

D. What is the domain of f (x) in part C above?

What is the domain of f ' (x) ?

The existence of a derivative at x = a depends on the existence of a limit at x = a .

$$f'(x) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

If the limit does not exist at x = a, we say the function *f* is **nondifferentiable at x = a,** or ***f '(a) does not exist.***

1. If the graph has a hole or a break at x = a, then f '(a) does not exist.
2. If the graph of *f* has a sharp corner at x = a, then f '(a) does not exist.
3. If the graph of *f* has a vertical tangent line at x = a, f '(a) does not exist.