Math142 Lecture Notes
3.5 - Basic Differentiation Properties

Constant Function Rule: First, remember that taking the derivative is going to tell us the slope of the *curve*. If we consider the graph of $f(x) = c$, a constant, what is the slope of the line? Right! It's zero. So the derivative of $f(x) = 9$ is zero. The derivative of $f(x) = \pi$ is $f'(x) = 0$.

Function $f(x)$	**Derivative** denoted $f'(x)$, y', $\frac{dy}{dx}$, *or* $\frac{d}{dx}[f(x)]$
x^2	$2x$
x^3	$3x^2$
x^5	$5x^4$
$2x$	2

Power Rule
If $f(x) = x^n$, where n is any real number, then $f'(x) = nx^{n-1}$.

Example 1: Differentiate each of the following.

(a) $f(x) = x^4$ (b) $g(x) = x^{1.32}$ (c) $y = \sqrt{x}$ (d) $h(x) = \dfrac{1}{x^3}$

Function	**Derivative**
$3x^2$	$6x$
$5x^5$	$25x^4$

Constant Multiple Rule
If $f(x) = kx^n$, where k and n are real numbers, then $f'(x) = n \cdot kx^{n-1}$

Example 2: Differentiate each of the following.

(a) $f(x) = 4x^3$ (b) $g(x) = \dfrac{2}{x^4}$ (c) $h(x) = \dfrac{2}{3}\sqrt[5]{x^2}$

Constant Function Rule
For any constant k, if $f(x) = k$, then $f'(x) = 0$.

Sum and Difference Rule
If $h(x) = f(x) \pm g(x)$, where f and g are both differentiable functions, then

$$h'(x) = f'(x) \pm g'(x)$$

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Example 3: Compute the following derivatives.

(a) $y = -8x^3 - 8x + 3$

(b) $f(x) = 5\sqrt{x} - \frac{1}{2}x + 7x^2$

(c) $g(x) = 4x^{-3/2} - 2x^{1/2} + x^{-1/2}$

(d) $F(x) = \dfrac{2x^3 + 3x^2 - x + 3}{x}$

(e) $y = \dfrac{x^3 + 2x^2}{2\sqrt[3]{x^2}}$

Example 4: Find the equation of the tangent line to $f(x)$ at $x = 1$.

(a) $f(x) = x^3 - 5x^2 + 8$

(b) $f(x) = 4x^2 - 5x + 2$

Example 5: If a coconut falls from a tree that is 75 feet tall, its height above the ground after t seconds is given by the *position function* $s(t) = 75 - 16t^2$ where $s(t)$ is measured in feet.

(a) The derivative of a position function is called the *velocity function.* It is denoted $s'(t) = v(t)$. Find $v(t)$.

(b) Compute $s(2)$ and $v(2)$ and interpret each.

(c) When does the coconut hit the ground?

Example 6: The Cool Air Company has determined that the total cost function for the production of its refrigerators can be described by $C(x) = 2x^2 + 15x + 1500$, where x is the weekly production of refrigerators and $C(x)$ is the total cost in dollars. The revenue function for these refrigerators is given by $R(x) = -0.3x^2 + 460x$ where x is the number of refrigerators sold and $R(x)$ is in dollars.

(a) Determine the profit function, $P(x)$, and the marginal profit function, $P'(x)$.

(b) Evaluate $P(20)$ and $P'(20)$ and interpret each.