Math 142 Lecture Notes
Section 4.1 – The Constant e and Continuous Compound Interest

Definition: The Number *e*

$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$. and is approximately 2.718281828459…

Note: The number *e* is an irrational number. An alternate form of the definition is:

$e = \lim_{s\to 0}(1+s)^{1/s}$

definition of "e"

☆**Continuous Compound Interest:**

$A = P \cdot e^{rt}$ memorize

where A = the amount accumulated at time t,
P= the principal amount of money,
r = the annual interest rate (expressed as a decimal),
t = the time expressed in years.

Sample problems:

1. If $3500 is invested at 8.25% compounded continuously for 3 years, how much to the nearest cent will be in the account?

P = 3500
r = 8.25%
t = 3

$A = Pe^{rt}$
$A = 3500e^{[0.0825(3)]}$
$A = 4482.867768$

$4482.87

2. If \$400 is invested at $5\frac{3}{4}\%$ compounded continuously for 6 months, how much to the nearest cent will be in the account?

$P = 400$

$r = 5\frac{3}{4}\%$

$t = 6 \text{ months} = \frac{1}{2} \text{ year}$

$A = Pe^{rt}$

$A = 400e^{(0.0575 * \frac{1}{2})}$

$A = 411.6669082$

\$411.67

3. How long will it take an investment of $1200 to grow to $5000, if invested at 7% compounded continuously?

$P = 1200$

$A = 5000$

$r = 7\%$

$t =$ **20.39 years**

$A = Pe^{rt}$

$5000 = 1200e^{0.07t}$

$\frac{5000}{1200} = e^{0.07t}$

$\ln\left(\frac{5000}{1200}\right) = \ln e^{0.07t}$

$\ln\left(\frac{5000}{1200}\right) = 0.07t$

$\frac{\ln\left(\frac{25}{6}\right)}{0.07} = t$ exact answer

$t \approx 20.38737651$

4. Which is the better option to borrow $24,000 to buy a new truck?

   A. Bank A which offers $7\frac{1}{4}\%$ interest compounded continuously for six years, or

   B. Bank B which offers $6\frac{1}{2}\%$ interest compounded semiannually for seven years.

A. $A = Pe^{rt}$

$A = 24{,}000e^{[0.0725(6)]}$

$A = \$37079.11$

B. TVM Solver

N = 2(7)

I% = 6.5

PV = -24000

PMT = 0

FV = ☐ Solve

P/Y = 2

C/Y = 2

FV = $37,555.37

Which value would I rather pay back?

Bank A: $37079.11 because the total cost is lower.