

Math142 Lecture Notes

4.2 - Derivatives of Exponential and Logarithmic Functions

Derivative Rules for Exponential Functions

- The derivative of $f(x) = e^x$ is $f'(x) = e^x$

Example 1: Differentiate and simplify the following.

(a) $f(x) = 4e^x + 8x^2 + 7x - 14$

(b) $g(x) = x^7 - x^5 + e^3 - x + e^x$

★ Changing from logarithmic notation to exponential form

Logarithmic form	Exponential form
$y = \log_b x$	$x = b^y$
Domain:	Domain:
Range:	Range:

The most common bases used in logarithms, are base 10 and base e .

Base 10: $\log x = \log_{10} x$

Base e : $\ln x = \log_e x$

★ Logarithmic Properties

It will be very useful to simplify logarithmic statements before taking the derivative.

$$\ln \frac{A}{B} = \ln A - \ln B$$

$$\ln A^p = p \cdot \ln A$$

Derivatives of Logarithmic Functions

- For $f(x) = \ln x$, with $x > 0$, $f'(x) = \frac{d}{dx}(\ln x) = \frac{1}{x}$.

Example 2: Differentiate and simplify the following.

(a) $f(x) = 6e^x + 2 \ln x$

(b) $g(x) = x^3 - \ln x^3 + 4$

★When the Base is not e

Derivatives of Exponential and Logarithmic Functions

- For $b > 0, b \neq 1$,

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} b^x = b^x \cdot \ln b$$

$$\frac{d}{dx} \ln x = \frac{1}{x}$$

$$\frac{d}{dx} \log_b x = \frac{1}{\ln b} \left(\frac{1}{x} \right)$$

★Change of Base Formula

$$b^x = e^{x \ln b}$$

Example 3: Change each of the following to base e :

a. 5^x

b. 10^x

Example 4: Find the derivative for each of the following functions:

a. $f(x) = 5^x - 10^x$

b. $g(x) = \log_2 x^9$

Example 5: An internet store sells a reversible fleece blanket. If the price-demand equation for reversible fleece blankets is $p = 200(.987)^x$, find the rate of change of price with respect to demand when the demand is 300 blankets and interpret.

Example 6: A model for newspaper circulation is $C(t) = 83 - 9 \ln t$ where $C(t)$ measures the newspaper circulation in millions of copies, and t represents the number of years since 1980. Use this model to estimate the circulation and the rate of change of circulation in the year 2010, rounded to the nearest tenth of a million copies.