Math142 Lecture Notes
4.3 - Derivatives of Products and Quotients

Product Rule
If $h(x) = f(x) \cdot g(x)$, where $f(x)$ and $g(x)$ are differentiable functions, then the derivative is $h'(x) = f(x)g'(x) + g(x)f'(x)$.

Example 1: Use the product rule to find the derivative for each of the following functions.

(a) $f(x) = x^4(2x^3 + 5x - 1)$

(b) $g(x) = (8x - 3)(x^2 - 6x + \pi)$

(c) $h(x) = (6\sqrt[3]{x^4} - 2\sqrt{x} + 3)\left(\frac{3}{x} - \frac{4}{x^2}\right)$

(d) Show two different ways to find the derivative of: $F(x) = 4x^3(2x^5 - 3x + 9)$

Example 2:

(a) Find the equation of the line tangent to $f(x) = (2x + 9)(x^2 - 12)$ at $x = -4$.

(b) Find the values of x for which the tangent line is horizontal.

Example 3: The monthly sales of a new computer are given by $q(t) = 30t - 0.5t^2$ units per month t months after it hits the market where $0 \leq t \leq 7$. The retail price, in dollars, is given by p(t) $= 2200 - 34t^2$ in t months after it hits the market, where $0 \leq t \leq 7$.

(a) Find $R(t)$ and $R'(t)$ but do not simplify.

(b) Find $R(6)$ and then interpret.

(c) Find the equation of the line tangent to $R(t)$ at $t = 6$

Quotient Rule

If $h(x) = \dfrac{f(x)}{g(x)}$, where f and g are differentiable functions with $g'(x) \neq 0$, then

$$h'(x) = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{[g(x)]^2}$$

*If $B = g(x)$ and $T = f(x)$ then the quotient rule becomes $h'(x) = \dfrac{B \cdot T' - T \cdot B'}{B^2}$

Example 4: Use the quotient rule to differentiate the following.

(a) $y = \dfrac{4x - 7}{3x - 5}$

(b) $f(x) = \dfrac{3\sqrt{x} + 8}{2x + 7}$

Example 5: The average hourly earnings, in dollars, of employees in finance, insurance, and real estate in the United States are given by

$$g(t) = 0.48t + 9.44 \quad 1 \leq t \leq 7$$

where t is the number of years since 1989. The average weekly earnings, in dollars, of employees in finance, insurance and real estate in the United States are given by

$$f(t) = 17.5t + 336.86, \quad 1 \leq t \leq 7$$

where t is the number of years since 1989.

(a) Let $h(t) = \dfrac{f(t)}{g(t)}$. What does this function represent?

(b) Compute $h(3)$ and interpret.

(c) Find $h'(t)$ and $h'(3)$ and interpret.

Example 6: Suppose that the concentration c of a drug in the blood stream t hours after it is taken orally is given by

$$c = \frac{4t}{t^2 + 3}$$

After two and a half hours, is the concentration increasing or decreasing? Explain.

Example 7: The value of a sport utility vehicle is given by the function

$$V(t) = \frac{50t + 350}{t^2 + 10}$$

when t is the number of years since purchased, and $V(t)$ is the value in thousands of dollars.

(a) What was the purchase price of the car?

(b) What will the value be in five years?

(c) Find the rate of change of the value of the vehicle.

(d) How fast is value changing after ten years?

Example 8: Given $f(3) = 4, f'(3) = -2, g(3) = -1$, and $g'(3) = 5$, find

(a) $h'(3)$ where $h(x) = f(x) \cdot g(x)$.

(b) $q'(3)$ where $q(x) = \dfrac{f(x)}{g(x)}$