

Math142 Lecture Notes

4.4 - The Chain Rule

Composite Functions

- A function m is a **composite** of functions f and g if $m(x) = f[g(x)]$.
- The domain of m is the set of all numbers x such that x is in the domain of g and $g(x)$ is in the domain of f .

Example 1: Find $f(g(x))$ and $g(f(x))$ for each of the following functions:

- Let $f(x) = 3x^2 + 5x$ and $g(x) = e^x$
- Let $f(x) = e^{2x}$ and $g(x) = x - 4$
- Let $f(x) = \frac{x+3}{x}$ and $g(x) = e^x$.

Example 2: Write each of the following functions as a composition of two simpler functions. Answers are not unique.

- $y = \sqrt{x^3 + 5}$
- $y = e^{5x^2 - 1}$
- $y = 48e^{-0.05x}$

Chain Rule

If $y = f(u)$ and $u = g(x)$ are used to define $h(x)$, where $h(x) = f(g(x))$,

$$\text{then } \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \quad \text{or} \quad h'(x) = f'(g(x)) \cdot g'(x)$$

Generalized Power Rule

If u is a differentiable function of x and n is any real number with $f(x) = [u(x)]^n$ then

$$f'(x) = n[u(x)]^{n-1} \cdot u'(x)$$

The Derivative of a Mess

If you have an expression of the form $y = (\text{mess})^n$, where n is any real number, the derivative is

$$y' = n(\text{mess})^{n-1}(\text{mess}')$$

Example 1: Differentiate the following functions.

(a) $f(x) = (8 - 3x)^5$

(b) $g(x) = 4(7x^2 - 5)^3$

(c) $f(x) = \frac{6}{x^3 - 4x + 8}$

(d) $f(x) = \frac{9}{\sqrt[3]{2x^3 + 4x}}$

Example 2: Find the following. You may need to use more than one derivative rule.

(a) Find $f'(x)$ if $f(x) = x(5 - 2x)^3$

(b) $\frac{d}{dx} \left(\frac{x^3}{(2x^3 - x + 4)^3} \right)$

(c) Find the equation of the line tangent to $g(x) = x^2\sqrt{3x^2 - 11}$ at $x=2$.

Example 3: During its first season, the number of viewers for *Cooking for Babies* can be modeled by

$$v(w) = (12 + 8w)^{\frac{2}{3}}$$

where w is the number of weeks since the show began, and v is the number of viewers in hundred thousands. Evaluate and interpret $v'(10)$

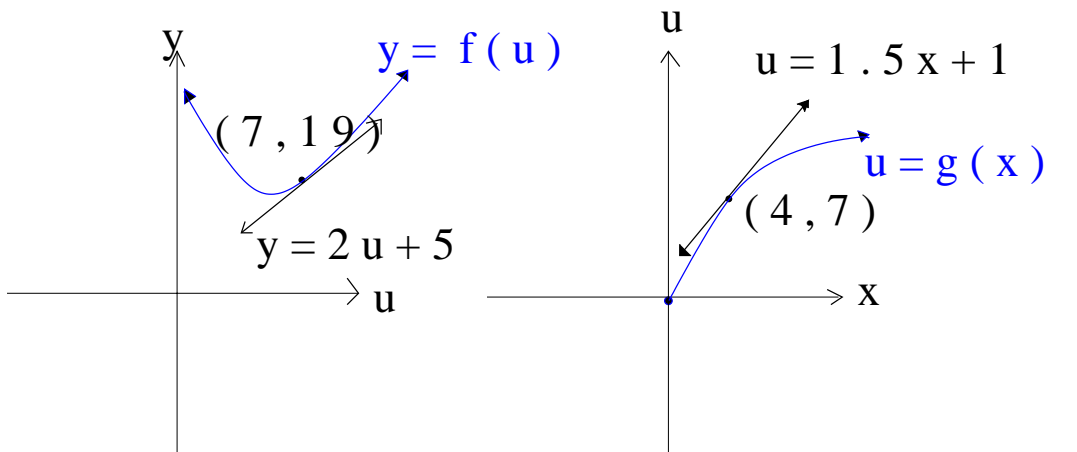
Example 4: Find $\frac{dy}{du}$, $\frac{du}{dx}$, and $\frac{dy}{dx}$ for each of the following:

(a) $y = u^{3/4}$ and $u = 4x^2 + 3$

(b) $y = e^u$ and $u = 5x^2 - 1$

(c) $y = \ln u$ and $u = 2x^3 + 8$

Example 5: Let $m(x) = f[g(x)]$. Use the chain rule and the graphs below to find each of the following:



(a) $f(7)$

(b) $g(4)$

(c) $m(4)$

(d) $f'(7)$

(e) $g'(4)$

(f) $m'(4)$

Example 6: Find the derivative of each of the following:

(a) $y = e^{5x+1}$

(b) $y = \ln(x^2 + 10)^4$

(c) $y = (e^{x^2} + 8)^3$