

## Math142 Lecture Notes

### 4.7 Elasticity of Demand

#### Definition: Relative and Percentage Rates of Change

- The **relative rate of change** of a function  $f(x)$  is  $\frac{f'(x)}{f(x)}$ .
- The **percentage rate of change** is  $100 \times \frac{f'(x)}{f(x)}$ .
- Note: Since  $\frac{d}{dx} \ln f(x) = \frac{f'(x)}{f(x)}$ , then the **relative rate of change of  $f(x)$**  is the derivative of the logarithm of  $f(x)$ .

Example 1: A broker is trying to sell you two stocks: Sunset Development and Aggie Growth Fund. The broker predicts the Sunset Development stocks will increase \$2 per year over the next few years, and Aggie Growth Fund will increase \$1 per year over the same time period. Which is the better buy?

Do you need additional information?

The cost of each stock is the missing information. Suppose the Sunset Development costs \$50 per share and Aggie Growth Fund costs \$20 per share.

Stock	Relative rate of change	Percentage rate of change
Sunset Development	$\frac{2}{50} = .04$	4%
Aggie Growth Fund	$\frac{1}{20} = .05$	5%

Example 2: Given the model for the population data is  $f(t) = 2.5t + 250$  where  $t$  is the number of years since 1990. Find the relative rate of change of  $f(t)$  on the interval  $[5, 12]$ .

Year	Real GDP in billions of dollars	Population in millions
1995	\$7,540	262.765
1996	\$7,810	265.190
1997	\$8,150	267.744
1998	\$8,500	270.299
1999	\$8,850	272.820
2000	\$9,190	275.306
2001	\$9,210	277.803
2002	\$9,440	280.306

Elasticity of demand is a mathematical tool that can be used to measure the impact that a change in price has on the demand for a product. The term **elasticity** generally refers to how sensitive the demand is to a change in price.

### Definition of Terms

- **Elastic:** a small change in price produces a significant change in demand.

$$E(p) > 1$$

- **Inelastic:** a change in price does not affect demand.

$$E(p) < 1$$

- **Unit:** a change in price produces the same change in demand.

$$E(p) = 1$$

### Elasticity of Demand

If price and demand are related by  $x = f(p)$ , then the elasticity of demand is given by  $E(p) = -\frac{p \cdot f'(p)}{f(p)}$

Suppose the price \$p, and quantity  $x$  are related by the equation  $x + 250p = 5275$ . In working with problems involving revenue, cost or profit, we usually use the demand equation, in other words, solve for  $x$ .

$$\begin{aligned}x + 250p &= 2000 \\x &= -250p + 2000 \\f(p) &= -250p + 2000\end{aligned}$$

Elasticity of demand is the negative of the ratio between relative rate of change of demand and the relative rate of change of price.

$$E(p) = - \frac{\text{relative rate of change of demand}}{\text{relative rate of change of price}}$$

$$E(p) = -\frac{p \cdot f'(p)}{f(p)}$$

**Elasticity of Demand**

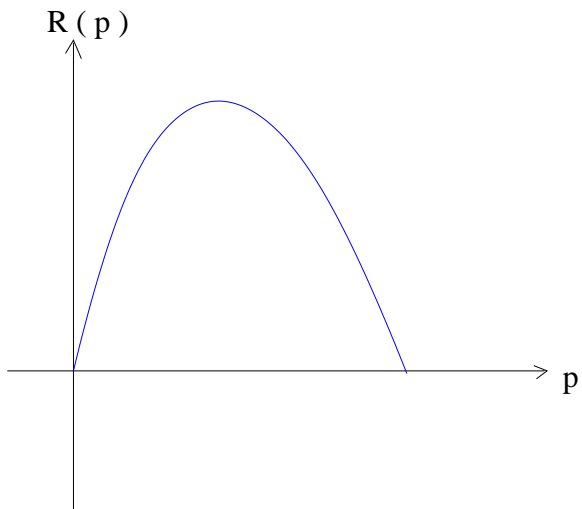
Given the demand function  $x = d(p)$ , the elasticity of demand, denoted  $E(p)$ , of a product at a price  $p$  is given by

$$E(p) = \frac{-p \cdot f'(p)}{f(p)}$$

$E(p)$ value	demand is:	change in price will cause:	$R'(p)$ is:
Between 0 and 1	Inelastic	relatively small changes in demand	positive $\rightarrow$ raise price.
Greater than 1	Elastic	relatively large changes in demand	negative $\rightarrow$ lower price.
Equal to 1	Unitary	relatively equal change in demand	zero $\rightarrow$ keep price same

Example 3: A school's junior business club is holding its annual raffle. Data collected from raffles in the past indicate that the demand function for the tickets follows the model  $p = \sqrt{36 - x}$  where  $p$  represents the price of a ticket and  $x$  represents the number of tickets each member sells each day.

- (a) Solve to get the demand function in the form  $x = f(p)$
- (b) Find the elasticity of demand  $E(p)$ .
- (c) Evaluate  $E(3)$ , interpret the results, and determine any adjustments the business club should make to the price of the tickets.
- (d) Evaluate  $E(4)$ , interpret the results, and determine any adjustments the business club should make to the price of the tickets.



Example 4: Find  $E(p)$  for the price-demand equation  $x = f(p) = 10,000 - 500p$  and interpret each of the following.

- $E(4)$

- $E(16)$

- $E(10)$