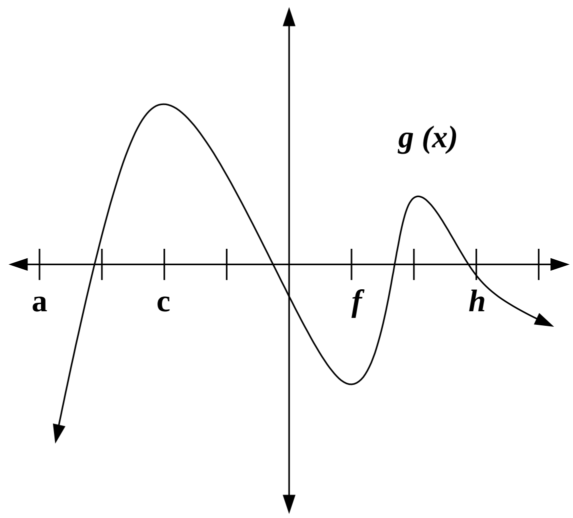
Math142 Lecture Notes
5.1 - First Derivatives and Graphs



$g(x)$ is . . .	Interval
increasing	
decreasing	
constant	

The slope of the tangent lines are	Interval
positive	
negative	
zero	

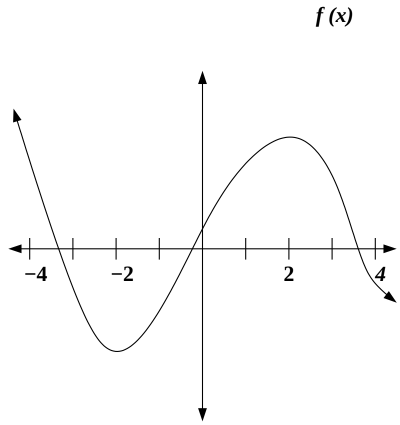
CONCLUSIONS:

1. $g(x)$ is increasing when ______________________________

2. $g(x)$ is decreasing when ______________________________

3. $g(x)$ is constant when ______________________________

Example 1: Use the graph of $f(x)$ below to fill in the chart.

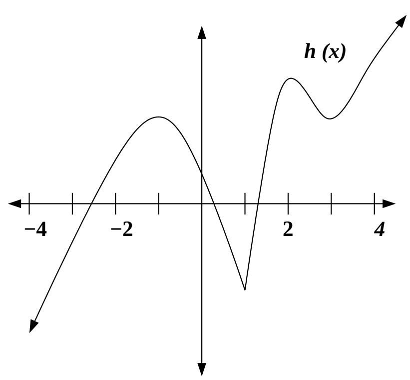


	intervals	what does this say about $f(x)$?
$f'(x) > 0$		
$f'(x) < 0$		
$f'(x) = 0$		

Relative Extrema (Relative maximums and minimums)
If f is a continuous function on an open interval containing c, the the point $(c, f(c))$:

- Is a relative maximum if $f(c) \geq f(x)$ for all x in the interval. (in other words $f(c)$ is the largest y-value when compared to y-values near it.)
- Is a relative minimum if $f(c) \leq f(x)$ for all x in the interval. (in other words $f(c)$ is the smallest y-value when compared to y-values near it.)

Example 2: Find all relative extrema in the graph of $h(x)$ below, then describe the behavior of $h'(x)$ at each extrema.



Critical Value
A critical value for f is an x-value **in the domain** of f for which

1. $f'(x) = 0$ **or**
2. $f'(x)$ is undefined

Example 3: Find the critical values for

(a) $f(x) = -x^3 - 3x^2 + 45x - 5$

(b) $g(x) = \dfrac{2-x}{x+5}$

(c) $h(x) = 8\ln x - x^2$

First Derivative Test Using a Sign Chart

Let $x = c$ be a critical value of a function f that is continuous on an open interval containing $x = c$. To determine whether $x = c$ is a relative maximum or minimum

1. Place c on a number line.
2. Choose a value that is less than c.
3. Substitute the chosen value into the first derivative.
4. Record the sign (+ or -) of the value found in (3) on the number line chart.
5. Repeat the process for a value greater than c.

Example 4: Determine the intervals where the following functions increase and decrease; then locate all relative extrema.

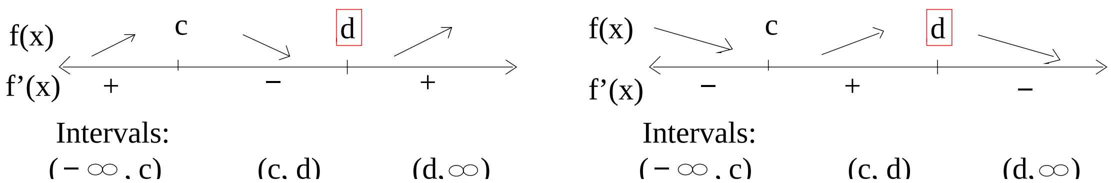
(a) $f(x) = -x^3 - 3x^2 + 45x - 5$

(b) $f(x) = \sqrt{2x - 8}$

Using a Sign Chart with Values Not in the Domain

Let $x = c$ be a critical value of a function f, and let $x = d$ be a value not in the domain of $f(x)$. Note: this function *is not continuous*. To determine where $f(x)$ has a relative minimum or maximum:

1. Place c and d on a number line.
2. Choose a value in each interval to the left and right of each point, c and d.
3. Substitute the chosen value into the first derivative.
4. Record the sign (+ or -) of the value found in (3) on the number line chart.
5. If the sign changes to the left and right of c, a critical point, you have a relative maximum or minimum.
6. If, however, the signs change to the left and right of d, you DO NOT have a relative maximum or minimum.



Example 5: Determine the intervals where the following functions increase and decrease; then locate all relative extrema.

(a) $f(x) = x + \dfrac{4}{x}$

(b) $g(x) = \dfrac{2x - 6}{x + 2}$