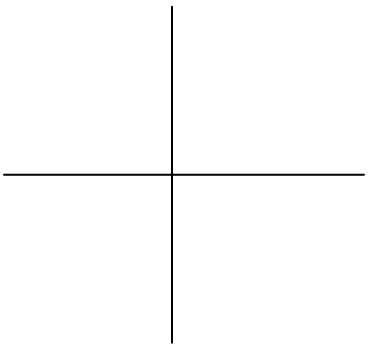
Math 142 Lecture Notes
Section 5.4 – Curve-Sketching Techniques

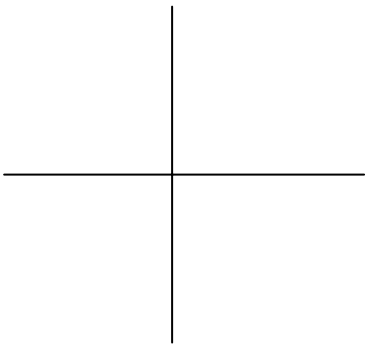
A Graphing Strategy

1. Analyze $f(x)$.
   - Find the domain of the function.
   - Find the intercepts.
   - Find the asymptotes
2. Analyze $f'(x)$.
   - Find the zeros of $f'(x)$.
   - Construct a sign chart for $f'(x)$, and determine where the function is increasing and decreasing.
   - Find the local max/mins.
3. Analyze $f''(x)$.
   - Find the zeros of f '' (x).
   - Using a sign chart, determine where the graph of the function is $\cup$ or $\cap$.
   - Find any points of inflection.
4. Sketch the graph of the function.

Example 1 : Given $f(x) = \dfrac{1-4x}{x^2}$

- Domain:
- Intercepts:
- Asymptotes:
- Intervals where f(x) is increasing:
- Intervals where f(x) is decreasing:
- Intervals where f(x) is concave up:
- Intervals where f(x) is concave down:





Example 2: Given $f(x)=\dfrac{2x^2+18}{4-x^2}$

- Domain:
- Intercepts:
- Asymptotes:
- Intervals where f(x) is increasing:
- Intervals where f(x) is decreasing:
- Intervals where f(x) is concave up:
- Intervals where f(x) is concave down:

Oblique Asymptotes: If the graph approaches a line that is neither horizontal nor vertical as x approaches infinity, or negative infinity, that line is called an oblique asymptote.

How to find Oblique Asymptotes: If the degree of the numerator is **one** more than the degree of the denominator, then the oblique asymptote can be found by polynomial long division.

Which of the following functions has an oblique asymptote? Find the oblique asymptote.

1. $f(x)=\dfrac{x^3-3x^2-28x}{x^4-16}$

2. $f(x)=\dfrac{x^3-3x^2-28x}{x^2-16}$

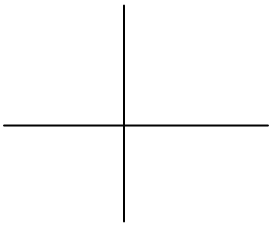
3. $f(x)=\dfrac{x^3-3x^2-28x}{x^3-27}$

4. $f(x)=2x+\dfrac{5}{x+1}$

Rational Functions - - Average Cost:

$C(x) = 5{,}000 + 0.5x^2$, where x is the number of items produced. Analyze the graph of the average cost function.

- Domain:
- Intercepts:
- Asymptotes:
- Intervals where f(x) is increasing:
- Intervals where f(x) is decreasing:
- Intervals where f(x) is concave up:
- Intervals where f(x) is concave down:



The minimum average cost occurs when the average cost is equal to the marginal cost.