

## Math 142 Lecture Notes Section 5.5 – Absolute Maxima and Minima

**Definition: Absolute Maxima and Minima**

If  $f(c) \geq f(x)$  for all  $x$  in the domain of  $f$ , then  $f(x)$  is called the absolute maximum value of the function  $f$ .

If  $f(c) \leq f(x)$  for all  $x$  in the domain of  $f$ , then  $f(x)$  is called the absolute minimum value of the function  $f$ .

**Look at the graphs of each of the following functions, and find relative max/min and absolute max/mins.**

1.  $f(x) = 0.2x^3 - 5x$

2.  $f(x) = 6 - 2x^2$

3.  $f(x) = x^{4/5}$

**★ Absolute Extrema:**

When asked to find the absolute extrema, find both absolute maximum, and the absolute minimum.

**Extreme Value Theorem:**

A function that is continuous on a closed interval  $[a, b]$  has both an absolute maximum value and an absolute minimum value on that interval.

**Use your calculator to look at the graphs of the following function over each of these intervals:**

**Set your window to: Xmin = -2, Xmax = 20, and Ymin = -10, Ymax = 240, Yscl=10.**

$$f(x) = x^3 - 21x^2 + 135x - 170$$

a) On the interval  $[2, 12]$

b) On the interval  $[4, 10]$

c) On the interval  $[4, 8]$

★ **Absolute Extrema:** Absolute extrema (if they exist), must occur at either critical values within the interval, or at the endpoints of the interval.

★ **To Find the Absolute Extrema:**

- Check to see if the function is continuous.
- Find the critical values in the interval (a,b).
- Evaluate the function at the endpoints, and at the critical values.
- The absolute maximum is the largest of the values.
- The absolute minimum is the smallest value found.

**Example:** Find the absolute maximum and absolute minimum values of

$f(x) = x^3 - 12x$  on each of the following intervals:

1. [-5,5]
2. [-3,3]
3. [-3,1]

**Second Derivative Test**

Let  $c$  be a critical value for  $f(x)$ .

$f'(c)$	$f''(c)$	Graph of $f(x)$	$f(c)$	Example
0	+	Concave upward	Local min	
0	-	Concave downward	Local max	
0	0	????	Test does not apply	

**Find the local maxima and minima for each function.**

**Use the second-derivative test when it applies.**

1.  $f(x) = x^3 - 9x^2 + 24x - 10$

2.  $f(x) = 10x^6 - 24x^5 + 15x^4$

★ **Finding an Absolute Extremum on an Open Interval**

Find the absolute maximum value of  $f(x) = 12 - x - \frac{9}{x}$  on the interval  $(0, \infty)$ .

Find the absolute maximum value of:  $g(x) = 4\ln(x) - 2x$  over its domain.