

Math142 Lecture Notes

6.1 - Antiderivatives and Indefinite Integrals

Find all possible functions $F(x)$ whose derivative is $f(x) = 2x$.

- $F(x) = x^2$
- $F(x) = x^2 + 5$
- $F(x) = x^2 + \pi$
- $F(x) = x^2 + C$

Definition: Antiderivative

- If $F'(x) = f(x)$, then $F(x)$ is called an antiderivative of $f(x)$.
- If C is any real number constant, then the general antiderivative of f on an interval is $F(x) + C$ if $\frac{d}{dx}[F(x) + C] = f(x)$ for all x in the interval. We use the notation

$$\int f(x)dx = F(x) + C$$

to denote $F(x) + C$ as the general antiderivative of $f(x)$.

This is read "The integral of $f(x)$ with respect to x is $F(x) + C$ "

Example 1: Determine if the function F is the general antiderivative of the function f .

(a) $F(x) = \frac{2}{3}x^{3/2} + 4x + C$; $f(x) = \sqrt{x} + 4$

(b) $F(x) = 2x^4 - x + C$; $f(x) = \frac{2}{3}x^3 - 1$

Power Rule for Integration

For any real number n , where $n \neq -1$, the indefinite integral of x^n is

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + C$$

Example 2: Determine the following indefinite integrals.

(a) $\int x^9 dx$

(b) $\int \frac{1}{t^{11}} dt$

(c) $\int \sqrt{3} y^2 dy$

Other Rules for Integration

- **Constant Rule**

If k is any real number, then the indefinite integral of k is $\int k dx = kx + C$.

- **Sum and Difference Rule**

For integrable functions f and g , $\int [f(x) \pm g(x)] dx = \int f(x) dx \pm \int g(x) dx$.

- **Coefficient Rule**

Given any real number coefficient c and integrable function f ,

$$\int c \cdot f(x) dx = c \cdot \int f(x) dx.$$

Example 3: Find the following.

(a) $\int 0.5 x^7 dx$

(b) $\int (5x - 2) dx$

$$(c) \int (4t^4 + 5t - 6) dt$$

$$(d) \int (x^{5/2} - \frac{4}{x^5} - \sqrt{x}) dx$$

$$(e) \int \frac{3y^2 - 2y}{6y} dy$$

Example 4: The Best Dressed Clothing Company finds that its marginal profit, MP , is linear and has the form $MP(q) = mq + b$, where m, b are constants. The company gets about \$171 additional profit from producing the 101st sport coat and \$169 additional profit from producing the 151st sport coat in each production run.

(a) Determine the marginal profit function MP .

(b) Knowing that the company gets \$11,300 profit from 150 sport coats, find the profit function P .

Indefinite Integrals of Basic Functions• **Power Rule**

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

• **Rule for e^x**

$$\int e^x dx = e^x + C$$

• **Rule for Logarithmic Functions**

$$\int \frac{1}{x} dx = \ln |x| + C, \quad x \neq 0$$

Example 5: Determine the following indefinite integrals.

(a) $\int \left(\frac{5}{x} - 8e^x \right) dx$

(b) $\int \frac{x^4 - 5x^2}{x^5} dx$

(c) $\int \left(\frac{4}{v} + \frac{v}{4} \right) dv$

Example 6: If the marginal cost of producing x units is given by $C'(x) = 0.9x^2 + 5x$ and the fixed cost is \$5000, find the cost function $C(x)$ and the cost of producing 25 units.