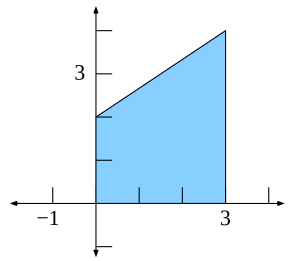
Math142 Lecture Notes
6-5 - The Fundamental Theorem of Calculus

We know from geometry that the area under the curve below is 9 square units.



In addition, we set up the definite integral $\int_0^3 (\frac{2}{3}x + 2)\ dx$ to represent the area. Now lets find the antiderivative $F(x)$ of $f(x) = \frac{2}{3}x + 2$ and then evaluate $F(3) - F(0)$.

The Fundamental Theorem of Calculus
If f is a continuous function defined on a closed interval $[a, b]$ and F is an antiderivative of f, then

$$\int_a^b f(x)\ dx = F(x)\ \Big]_a^b = F(b) - F(a)$$

Example 1: Evaluate $\int_3^6 (4x - 2)\ dx$

Example 2: Find the exact area under $f(x) = x^2 - 2x + 3$ on $[1, 3]$.

Example 3: The daily cost function for Playa del Carmen Cruises is given by the function: $C(x) = -2.5x^2 + 430x + 432.50$, over the interval $0 \leq x \leq 140$

(a) Calculate the change in cost from $x = 21$ to $x = 33$.

(b) Graph the marginal cost function and use geometric formulas to find the area between $C'(x)$ and the x-axis from $x = 21$ to $x = 33$.

(c) Compare your results.

(d) Evaluate the definite integral: $\int_{21}^{33} C'(x)\, dx$

Example 4: The Tiny Tot Toy Company determines that the marginal cost for producing a new action figure is given by

$$MC(x) = C'(x) = 4 - 0.02x, \quad 0 \leq x \leq 100$$

where x is the number of toys made daily and $MC(x)$ is the marginal cost measured in dollars per toy.

(a) Evaluate $MC(30)$ and interpret.

(b) Evaluate $\int_{0}^{30} MC(x)\, dx$ and interpret.

Example 5: Finding distance traveled given velocity functions

The velocity of an object can be modeled by $v(t) = 12t + 40$ where t is the time in seconds and $v(t)$ is the velocity measured in $\frac{feet}{second}$.

a. Find the distance traveled between 5 seconds and 25 seconds.

b. Evaluate $v(10)$ and interpret.

Example 6: If the rate of change of sales of an item is given by $S'(t) = 9t^2 + 18t$ where t is the number of weeks after an advertising campaign has begun, how many items were sold during the third week?

Example 7: Definite Integrals and Substitution Techniques

- Evaluate: $\displaystyle\int_0^5 \frac{x}{x^2+10}\,dx$

- Evaluate: $\displaystyle\int_2^5 \frac{1}{\sqrt{6-t}}\,dt$

- Evaluate: $\displaystyle\int_0^2 e^x\,dx$

- Evaluate to three decimal places: $\displaystyle\int_{1.5}^{4.3} \frac{x}{\ln x}\,dx$

Average Value of a Continuous Function f over the interval [a,b]

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

Example 8: Find the average value of $f(x) = 6t^2 - 2t$ over the interval $[-2, 3]$.

Example 9: Find the average price (in dollars) given $p = D(x) = 100e^{-0.05x}$ over the interval $[40, 60]$.