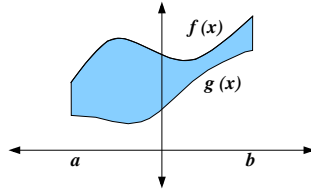


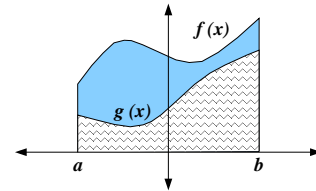
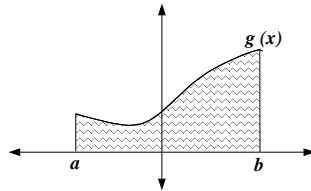
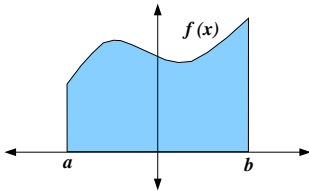
## Math142 Lecture Notes

### 7-1 - Area between Curves

How can we find the area of the shaded region below?



We can find the area under  $f(x)$ , the area under  $g(x)$ , and then subtract the area under  $g(x)$  from the area under  $f(x)$  to find the area between  $f(x)$  and  $g(x)$ .



#### Area Between Two Curves

On a closed interval  $[a, b]$ , the area between two continuous functions  $f(x)$  and  $g(x)$  where  $f(x) \geq g(x)$  is given by

$$\text{Area} = \int_a^b [f(x) - g(x)] dx$$

This can also be thought of as  $\int_a^b [\text{top function} - \text{bottom function}] dx$

Example 1: Determine the area between  $k(x) = -x^2 + 4$  and  $h(x) = 2x + 7$  on  $[-3, 2]$ .

Example 2: Determine the area bounded by the curves  $f(x) = -x - 3$  and  $g(x) = 9 - x^2$ .

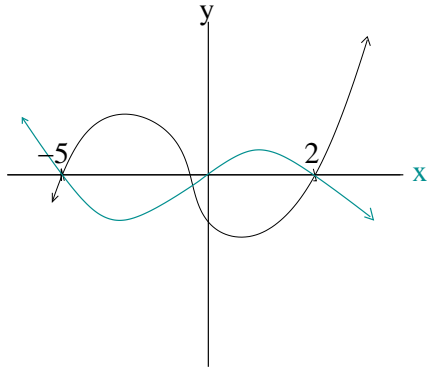
Example 3: Determine the area bounded by the curves  $f(x) = 2x^3$  and  $g(x) = -x^3 + x^2 + 2x$ .

Example 4: Find the area between the graph of  $f(x) = x^2 - 9$  and the  $x$  axis over the indicated intervals.

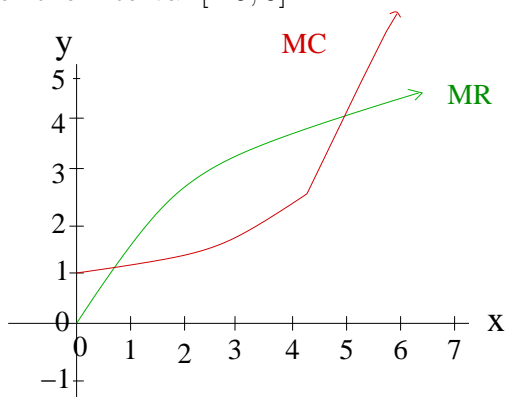
a.  $[0, 2]$

b.  $[2, 4]$

Example 5: A market analyst for Chocolate Time estimates that with no promotion the annual sales of its candy bar, Chococoloco Dream, can be modeled by  $s_1(t) = 0.76t + 8.42$  million dollars per year,  $t$  years from now. This same analyst estimates that, with a modest promotional campaign, annual sales can be modeled by  $s_2(t) = 11.4e^{0.05t}$ . During the first 5 years, what will the total increase in sales be in response to the promotion?



Example 6: Find the area between  $y_1 = x^3 + 4x^2 - 7x - 10$  and  $y_2 = -2x^3 - 6x^2 + 20x$  over the interval  $[-5, 5]$ .



Example 7: The marginal revenue function and marginal cost function for Creative Cruises, are shown above, with  $x$  representing number of hundreds of passengers, and  $y$  representing millions of dollars. Write an integral which represents the profit function for Creative Cruises next trip on a ship which holds six-hundred passengers.

Example 8: Find the area (to three decimal places) bounded by the graphs of  $f(x) = x^2 \cdot \ln x$  and  $g(x) = 3x - 3$ .

Example 9: Find the area to three decimal places bounded by the graphs of  $f(x) = e^{-x^2}$  and  $g(x) = x^2 - 1$ .

Example 10: *Distribution of wealth*: The data in the table describes the distribution of wealth in a country:

$x$	0	0.20	0.40	0.60	0.80	1
$y$	0	0.12	0.31	0.54	0.78	1

Use quadratic regression to find the equation of a Lorenz curve for the data.

Use the regression equation and a numerical integration routine to approximate the Gini index of income concentration.

Source: Calculus by Barnett, Ziegler and Byleen, p. 433