Math142 Lecture Notes
8.1 - Functions of Several Variables

Function of Two Independent Variables
An equation of the form $z = f(x, y)$ represents a function of two independent variables if, for each ordered pair of real numbers (x, y), the equation determines a unique real number z.

Example 1: Given $f(x, y) = 4x^2 - 3xy + \sqrt{y}$, determine each of the following:

(a) $f(2, 4)$

(b) $f(-3, 1)$

(c) $f(-1, 9)$

Example 2: Playgrounds Plus offers two models of playground equipment for backyard use. Model A contains two swings and a slide and cost $125 to build. Model B contains two swings, a trapeze bar, and a slide and cost $140 to build. Write a cost function for Playground Plus, if x represents the number of Model A units built, and y represents the number of Model B units built.

Example 3: If they can sell all those they build at the following prices, Model A for $210, and Model B for $235, write a revenue function for Playground Plus.

Example 4: Write a profit function for Playground Plus.

Examples of Functions of Several Variables

- Area of a rectangle: $A(x, y) = x \cdot y$
- Volume of a box: $V(x, y, z) = x \cdot y \cdot z$
- Volume of a right circular cylinder: $V(r, h) = \pi r^2 h$
- IQ: $Q(M, C) = \dfrac{M}{C}(100)$, M = mental age, C= chronological age
- Volume of a cone: $V(r, h) = \dfrac{1}{3}r^2 h$
- Volume of a pyramid: $V(h, a) = \dfrac{1}{3}ha^2$, h= ht of the pyramid, a=length of a side on the square base
- The quadratic formula $(ax^2 + bx + c = 0)$: $f(a, b, c) = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Surface Area of a Right Rectangular Solid: $S_A(x, y, z) = 2xy + 2yz + 2xz$
- **Cobb-Douglas production function**: $f(x, y) = kx^m y^n$, $m + n = 1$

Example 5: A company uses an open box with a square base for one of its products. If x inches is the length of one side of the base, and the height is y inches, find each of the following:

- Surface Area, $S_A(x, y) =$

- $S_A(3, 5)$ and interpret your answer.

Example 6: The Cobb-Douglass production function $f(x, y) = kx^m y^n, m+n = 1, k, m, n > 0$ is used to describe the number of units produced from the utilization of x units of labor, and y units of capital. For example, the productivity of Playful Playgrounds is defined by $f(x, y) = 35x^{0.3}y^{0.7}$, where x is the units of labor, and y is the units of capital. How many units of playgrounds will be produced if the company uses 120 units of labor and 500 units of capital.?

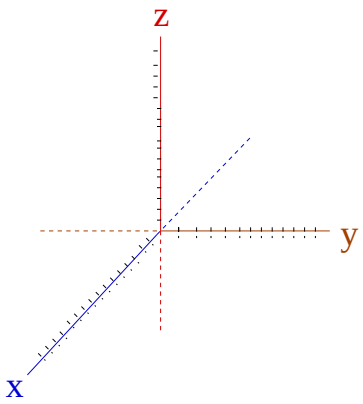
Plotting in Three Space

- (x, y, z) The x-, y-, and z-axis are all perpendicular.
- From the origin, move x units along the x-axis, then
- y units parallel to the y-axis, then
- z units parallel to the z-axis.

Example 7: Plot each of the following points:

a. $(2, 3, -1)$

b. $(5, -4, 2)$



Cobb-Douglas Production Function

- If L represents the units of labor and K represents the units of capital, then the total production Q is given by the Cobb-Douglass production function

$$Q = aL^bK^c$$

- Note: b and c add up to 1.

Example 8: A golf club manufacturer has a Cobb-Douglas production function given by

$$Q = f(x, y) = 21x^{0.3}y^{0.7}$$

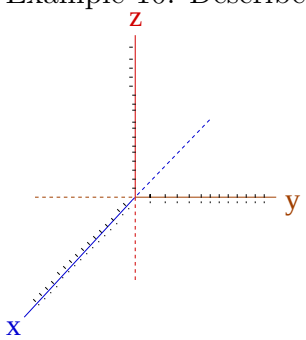
where x is the utilization of labor (in millions), y is the utilization of capital (in millions), and Q is the number of units of golf clubs produced.

(a) Compute $f(2, 5)$ and interpret.

(b) Compute $f(4, 2)$ and interpret.

Example 9: Describe the cross sections of $f(x, y) = y^2 - x^2$ in the planes $y = 0, y = 1, y = 2, y = 3$, and $y = 4$.

Example 10: Describe the graph of the equation $2x^2 + 2y^2 + 2z^2 = 50$.



Example 11: Find the domain of each of the following problems:

$f(x, y) = 4x^2 - 3xy^3$

$f(x, y) = \sqrt{3x - 2y}$

$f(x, y) = \dfrac{3x + 10}{x^2 - y^2}$

$f(x, y) = \sqrt[3]{2x - 3y^2 + 12y}$