

Math142 Lecture Notes

8.3 - Maxima and Minima

Definition of Relative Extrema in 3-Space

Let $f(x, y)$ be a function of two variables. The value $f(a, b)$ is:

1. A relative maximum if $f(a, b) \geq f(x, y)$ for all points (x, y) in some circular region in the xy -plane around (a, b) , where (a, b) is the center of the circular region.
2. A relative minimum if $f(a, b) \leq f(x, y)$ for all points (x, y) in some circular region in the xy -plane around (a, b) , where (a, b) is the center of the circular region.

Critical Point

The point (a, b) is a critical point for $f(x, y)$ if $f_x(a, b) = 0$ **AND** $f_y(a, b) = 0$

Example 1: Determine the critical points for $f(x, y) = x^2 + y^2 - xy + 3y$.

Second Derivative Test

Let $z = f(x, y)$ be a function of two variables such that $f_{xx}(x, y)$, $f_{yy}(x, y)$, and $f_{xy}(x, y)$ all exist. If $f_x(a, b) = 0$ and $f_y(a, b) = 0$ then we define the number D to be

$$D = f_{xx}(a, b) \cdot f_{yy}(a, b) - [f_{xy}(a, b)]^2$$

The Second Derivative Test has the following form:

1. If $D > 0$ and $f_{xx}(a, b) > 0$, then $f(a, b)$ is a relative minimum.
2. If $D > 0$ and $f_{xx}(a, b) < 0$, then $f(a, b)$ is a relative maximum.
3. If $D < 0$ then $f(a, b)$ is a saddle point.
4. If $D = 0$, the test gives no information about $f(a, b)$.

Example 2: Determine any relative extrema for $f(x, y) = x^2 + y^2 - xy + 3y$.

Example 3: Determine the relative extrema for $f(x, y) = x^3 + 3x^2 - y^2 + 2y + 4$.

Example 4: The All Clear Company sells two types of car windshield wiper fluid: *no-freeze*, which can be used at temperatures above -30°F , and a *regular*, which can be used at temperatures above 10°F . Let x represent the number of gallons of no-freeze sold each year, in millions, and let y represent the number of gallons of regular sold each year, also in millions. Suppose that

$p = 3.9 - 0.5x - 0.1y$, is the price in dollars of a gallon of no-freeze

$q = 3.9 - 0.1x - 0.8y$, is the price in dollars of a gallon of regular

(a) Determine the revenue function $R(x, y)$.

(b) Determine x and y such that revenue is maximized and find the maximum revenue.