Review for EXAM # 1

MATH 142 Drost-Spring 2010

1. Find the domain of $f(x) = \dfrac{\sqrt{9-3x}}{x^2-5}$ and write your answer in inequality notation.

2. Write a piece-wise defined function to represent the salary of sales personnel for Fine Furniture Wholesalers. They make \$1800/month plus 2% of sales over \$25,000/month but not more than \$40,000, and a super bonus of 8% of sales over \$40,000. Let s represent sales per month in dollars, and M represent monthly salary in dollars.

3. Find the domain of $g(x) = \dfrac{2x^2+x-3}{3x^2-x-2}$ and write your answer in interval form.

4. Find the average rate of change between the points $(8,-2)$ and $(-5,4)$.

5. Determine the x- and y-intercepts for the graph of $f(x) = \dfrac{75-10x}{5-x}$.

6. Given $f(x) = 2x - 3x^2$, use the difference quotient to determine the slope of the secant line where $x = -2$ and $\Delta x = h = 3$.

7. Given $f(x) = \dfrac{x^2}{2x+5}$

   a. Find the intercepts

   b. Find any horizontal asymptotes.

   c. Find any vertical asymptotes.

8. Classify the functions below as exponential growth or decay (in other words, is the function increasing or decreasing):

   $f(x) = (\frac{2}{5})^x$

   $g(x) = (\frac{3}{2})^{-x}$

   $h(x) = e^{0.7x}$

9. Kyle deposits \$2400 into an account that pays interest at a rate of 6.25% compounded weekly.

   a. How long before the account reaches \$4,000?

   b. How much is in the account after 3 years?

10. J.J. deposits \$1500 into an account paying $6\frac{3}{4}$% compounded continuously.

    a. How long before the balance is \$2500?

    b. What is the balance after 2 years?

11. Given $f(x) = 2.5e^{.02x}$ where x is the time in minutes and f(x) is the number of bacteria in the culture in thousands. Find the number of bacteria in the culture after 2 hrs, rounded to the nearest hundred.

12. Rewrite in logarithmic form:

    a. $10^x = 2.4$

    b. $5^{x+1} = 3$

    c. $e^{x^2} = 14$

13. $y^2 + 3x = 5$ Does this equation describe a function? Why, or why not?

14. Solve for x: $\log_7(\log(\ln x)) = 1$

15. Given $\log(x) = 8$ and $\log(y) = 12$, evaluate: $\log(1000x^2y)$

16. Solve for x: $5 \cdot 3^{2x-1} = 135$

17. Solve for x: $\log_4(x^2 - 9) = 2$

18. Solve for x: $\log_2(x+1) = 4 - \log_2(x-5)$

19. If planted with 100 trees, each tree produces \$50 per year in produce. Due to overcrowding, for each additional tree planted the yield drops 50 cents. How many trees should be planted to maximize the revenue?

    If the cost to care for the trees (fertilizer and water) run \$15 per year per tree, how many trees should be planted to maximize profit?

20. Find the following for $f(x) = \dfrac{x^2-3x-4}{2x^2+5x+3}$, if they exist.

    a. vertical asymptote(s):

    b. horizontal asymptote(s):

    c. hole(s) in the graph:

    d. x-intercept(s):

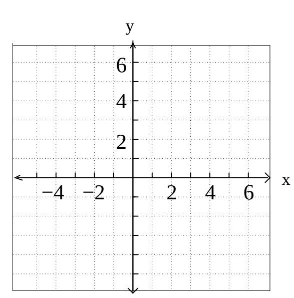
    e. y-intercept(s):

21. How long does it take for an account earning $10\frac{3}{4}$% compounded quarterly to double?

22. Find the present value of an account compounded continuously at $5\frac{1}{4}$% if in ten years it grows to \$8000.

23. Graph the piecewise defined function

$$f(x) = \begin{cases} |\, x-2 \,|, & \text{if } x < -2 \\ 5, & \text{if } -2 \le x \le 1 \\ x^2+1 & x > 2 \end{cases}$$



24. A machine purchased for \$1800 has a useful life of 6 years and a scrap value of \$500. Assuming a straight line depreciation, find its value at 4 years old.

25. Solve: $25^{3x} = 125^{x-4}$

26. Find the derivative of $f(x)$ given that $f(x+h) - f(x) = \dfrac{3}{2x+2h-5} - \dfrac{3}{2x-5}$.

27. Find the derivative using the limit definition of the derivative for

    a. $f(x) = \sqrt{4-3x}$.

    b. $g(x) = x^2 + 5x - 10$

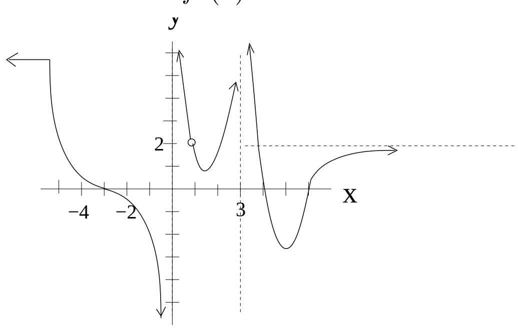
    c. $h(x) = \dfrac{x}{x+2}$

28. An object moves along the y- axis as described by $y = x^2 + 2x$, where x is the time in seconds, and y is measured in feet.

    a. Find the average velocity when x changes from 2 seconds to 5 seconds.

    b. Find the instantaneous velocity at x=2 seconds.

29. Refer to the graph of $f(x)$ below. List the values of x for which $f'(x)$ does not exist .



30. The profit in dollars from the sale of x Wii © consoles is given by $P(x) = 200x - 0.01x^2 - 3000$.

    a. Find the average change in profit if production changes from 7000 units to 8000 units.

    b. Find $P'(x)$.

    c. Find the instantaneous rate of change of profit at 7000 units.

    d. As the plant manager, would you keep production the same (7000/yr), increase, or decrease production?

31. How can the derivative be used to find the maximum and minimum?

32. Which of the following functions represent $1-1$ functions?

    a. $y = x^2 - 10x + 24$

    b. $y = |2x - 5|$

    c. $y = \ln(x-5) + 1$

    d. $y = \sqrt[3]{x-1}$

    e. $y = e^{x-2}$

33. Solve the following inequality using a sign chart.

$$\frac{2x-10}{x+4} \geq 0$$

34. Find

$$\lim_{x\to -3} \frac{2x|x+3|}{x+3}$$

35. Find

$$\lim_{x\to -2} \sqrt{3x^2+5x+2}$$

36. Find

$$\lim_{x\to 4} (2x^2 - 5x + 1)$$

37. Find $\lim_{x\to 2^+} f(x)$ *AND* $\lim_{x\to 2^-} f(x)$ when

$$f(x) = \begin{cases} |x-3|, & x < 2 \\ x^2 - 1, & x \geq 2 \end{cases}$$

38. Find: $\lim_{x\to\infty} \dfrac{3x-4}{x-1}$

39. Find: $\lim_{x\to -\infty} (6 - 2x - x^3)$

40. Find the inverse, y_2, of $y_1 = \ln(2x - 30)$.

    State the domain and range of y_1 and y_2.

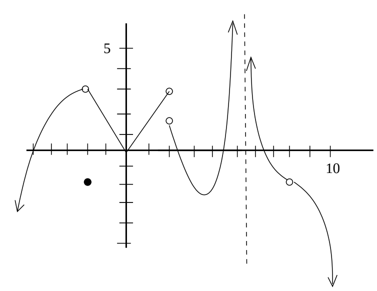
41. Find $\lim_{x\to 4} \dfrac{5x^2 - 28x + 32}{x^2 - 16}$

42. State the best reason why $f(x)$ shown below is not continuous at each point of discontinuity:

    i. $f(a)$ is not defined.

    ii. $\lim_{x\to a} f(x)$ is not defined.

    iii. $\lim_{x\to a} f(x) \neq f(a)$



43. The revenue from the sale of Valentine Candy is given by $R(x) = 40x - 0.25x^2$ over the interval $0 \leq x \leq 150$.

    a. What is the change in revenue if sales change from 50 boxes of Valentine candy to 80 boxes?

    b. Find the average change in revenue for part (a).

44. The derivative **does not exist** under what conditions?

45. a. Find the maximum revenue for *Hot Air Balloon Adventures* where x represents the number of rides provided weekly and $R(x)$ is measured in dollars. They have a price-demand equation of $p(x) = 100 - x$.

    b. How many rides must be scheduled to maximize the revenue?

    c. If the fixed costs for *Hot Air Balloon Adventures* is \$900/wk and the total costs to provide 10 rides is \$1150, find the break-even point(s).

46. The derivative of $f(x) = 4x^2 + 5x - 10$ using the power rule, and constant rule is:

47. Given $f(x) = 2x^2 - x + 1$

    a. find the slope of the tangent line at $x = 4$.

    b. write the equation of the tangent line.

48. Find the derivative, y', if

    a. $y = x^{-4} + x^{\frac{3}{2}} - e^3$.

    b. $y = \dfrac{1}{\sqrt[3]{x^2}}$.

    c. $y = 4.2x^{-2} - \dfrac{0.5}{\sqrt[4]{x}} + 2$.

    d. $y = x^2 - 1.5x - 10\sqrt{x}$.

    e. $y = \dfrac{x^5 - 5x^3 - 2}{x^2}$.

49. The price-demand function and the cost function for the production of air-conditioning units is $x = 2000 - 0.25p$ and $C(x) = 60,000 + 200x$.

    a. Find the average cost of making 100 units.

    b. Find the marginal cost of making 100 units.

    c. Find the marginal average cost when $x = 100$.

    d. Find the revenue when 100 units are made and sold.

    e. Find the average revenue when 100 units are made and sold.

    f. Find the revenue of making and selling 25 units.

    g. Find the approximate revenue from the 25^{th} unit.

    h. Find the marginal average revenue function.

    i. What is the profit from making and selling 100 units?

    j. What is the marginal profit function.

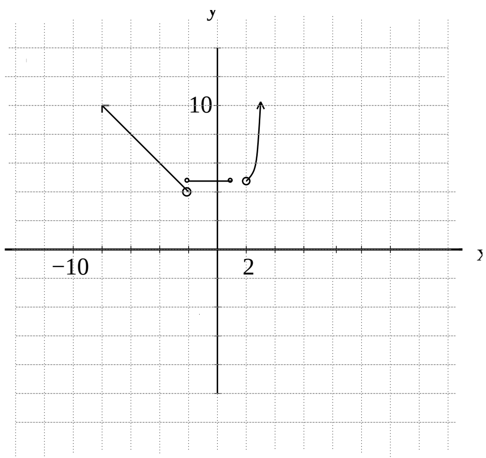
    k. Find the marginal average cost function.

    l. How many should they make and sell to maximize revenue?

    m. How many should they make and sell to maximize profit?

ANSWER KEY

1. $\Re, x \leq 3, x \neq \pm\sqrt{5}$
2. $M(s) = \begin{cases} 1800, & 0 \leq s \leq 25000 \\ 1300 + 0.02s, & 25000 < s \leq 40000 \\ .08s - 1100, & s > 40,000 \end{cases}$
3. Domain: $\Re, (-\infty, -\frac{2}{3}) \cup (-\frac{2}{3}, 1) \cup (1, \infty)$
4. $-\frac{6}{13}$
5. $(7.5, 0), (0, 15)$
6. 5
7. a. $(0, 0)$
   b. none
   c. $x = \dfrac{-5}{2}$
8. $f(x)$: decay
   $g(x)$: decay
   $h(x)$: growth
9. a. 8.2 yrs.
   b. \$2894.63
10. a. 7.6 yrs
    b. \$1836.69
11. $27,600$ bacteria
12. a. $\log(2.4) = x$
    b. $\log_5(3) = x + 1$
    c. $\ln(14) = x^2$
13. When you solve for y, you get two values, therefore it is not a function.
14. $x = e^{10^7}$
15. 31
16. $x = 2$
17. $x = 5, -5$
18. $x = 7$
19. 100 trees; 85 trees
20. a. vertical asymptote(s): $x = -1.5$
    b. horizontal asymptote(s): $y = 0.5$
    c. hole(s): at $x = -1$
    d. x-intercept(s): $(4, 0)$
    e. y-intercept(s): $(0, \frac{-4}{3})$
21. 6.5 yrs.
22. $\$4,732.44$.



23.

24. $V(4) = \$933.33$

25. $x = -4$

26. $f'(x) = \dfrac{-6}{(2x-5)^2}$

27. Find the derivative using the limit definition of the derivative for

   a. $f'(x) = \dfrac{-3}{2\sqrt{4-3x}}$

   b. $g'(x) = 2x + 5$

   c. $h'(x) = \dfrac{2}{(x+2)^2}$

28. a. 9 ft/sec; b. 4 ft/sec

29. $f'(x)$ does not exist at:

   a. corner $x = -5$

   b. vert. asym $x = 0, x = 3$

   c. hole $x = 1$

30. a. \$50/unit, b. $P'(x) = 200x - 0.02x$, c. $P'(7000) = \$60$/unit, d. Increase, because the derivative is positive (rate of change of profit is increasing).

31. Find where $f'(x) = 0$

32. c, d, and e

33. $(-\infty, -4) \cup [5, \infty)$

34. DNE

35. 2

36. 13

37. 3, 1

38. 3

39. ∞

40. $y_2 = \frac{1}{2}e^x + 15$

   y_1: D: $x > 15$, R: $\Re$

   y_2: D: $\Re$, R: $y > 15$

41. $\dfrac{3}{2}$

42. i. $x = 8$

   ii. $x = 5.5, x = 2$

   iii. $x = -2$

43. a. \$225, b. \$7.50/box

44. a. corner or cusp, b. vertical asymptote, c. hole

45. a. \$2500, b. 50 rides, c. $(15, 1275), (60, 2400)$

46. $f'(x) = 8x + 5$

47. a. 15, b. $y = 15x - 31$

48. a. $y' = \dfrac{-4}{x^5} + \dfrac{3}{2}x^{\frac{1}{2}}$

   b. $y' = \dfrac{-2}{3x^{\frac{5}{3}}}$

   c. $y' = \dfrac{-8.4}{x^3} + \dfrac{1}{8x^{\frac{5}{4}}}$

   d. $y' = 2x - 1.5 - \dfrac{5}{x^{\frac{1}{2}}}$

   e. $y' = 4x^3 - 10x + \dfrac{2}{x^2}$

49. a. \$800, b. 200, c. -6

   d. \$760,000, e. \$7,600, f. \$197,500

   g. \$7,808, h. -4, i. \$680,000,

   j. $MP = 7800 - 8x$ k. $MAC = \dfrac{-60,000}{x^2}$,

   l. $x = 1000$, m. $x = 975$