Review for EXAM # 1

MATH 142 Drost-Spring 2010

1. Find the domain of $f(x) = \dfrac{\sqrt{9-3x}}{x^2-5}$ and write your answer in inequality notation.

$9-3x \geq 0$ $\quad x^2-5 \neq 0$

$-3x \geq -9$ $\quad x^2 \neq 5$

$x \leq 3,$ $\quad x \neq \pm\sqrt{5}$

$\boxed{x \leq 3, x \neq \pm\sqrt{5}}$

2. Write a piece-wise defined function to represent the salary of sales personnel for Fine Furniture Wholesalers. They make \$1800/month plus 2% of sales over \$25,000/month but not more than \$40,000, and a super bonus of 8% of sales over \$40,000. Let s represent sales per month in dollars, and M represent monthly salary in dollars.

$$M(s) = \begin{cases} 1800, & 0 \le S \le 25{,}000 \\ 1800 + .02(S - 25{,}000), & 25000 < x \le 40{,}000 \\ 1800 + .02(15{,}000) + .08(S - 40{,}000), & x > 40{,}000 \end{cases}$$

$$M(s) = \begin{cases} 1800, & 0 \le S \le 25{,}000 \\ 0.02\,S + 1300, & 25000 < S \le 40000 \\ 0.08\,S - 1100, & S > 40{,}000 \end{cases}$$

3. Find the domain of $g(x) = \dfrac{2x^2 + x - 3}{3x^2 - x - 2}$ and write your answer in interval form.

$$g(x) = \frac{(2x+3)(x-1)}{(3x+2)(x-1)}$$

$-2/3$ $\quad$ 1

$(-\infty, -\frac{2}{3})$ $\quad$ $(-\frac{2}{3}, 1)$ $\quad$ $(1, \infty)$

$$\boxed{(-\infty, -\tfrac{2}{3}) \cup (-\tfrac{2}{3}, 1) \cup (1, \infty)}$$

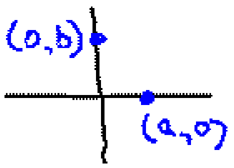
3

4. Find the average rate of change between the points $(8, -2)$ and $(-5, 4)$.

$$m = \frac{4-(-2)}{-5-8} = \boxed{\frac{6}{-13}}$$

4

5. Determine the x- and y-intercepts for the graph of $f(x) = \dfrac{75 - 10x}{5 - x}$.



x- intercept

Set numerator $= 0$

$75 - 10x = 0$

$75 = 10x$

$7.5 = x$ $\quad (7.5, 0)$

y- intercept

Substitute $x = 0$

$y = \dfrac{75-0}{5-0} = 15$ $\quad (0, 15)$

5

6. Given $f(x) = 2x - 3x^2$, use the difference quotient to determine the slope of the secant line where $x = -2$ and $\Delta x = h = 3$.

$f(x) = 2x - 3x^2$

$f(-2) = 2(-2) - 3(-2)^2 = -4 - 12 = -16$

$(-2, -16)$

$f(-2+3) = f(1) = 2 - 3 = -1$

$(1, -1)$

$m = \frac{-1-(-16)}{1-(-2)} = \frac{15}{3}$

$m = \boxed{5}$

6

7. Given $f(x) = \dfrac{x^2}{2x+5}$

a. Find the intercepts

b. Find any horizontal asymptotes.

c. Find any vertical asymptotes.

a. x - intercept (0,0)

Set numerator = 0

y - intercept

let $x = 0$ $y = \frac{0}{5} = 0$ (0,0)

b. none (the degree of the numerator is greater than the degree of the denominator.

c. $x = -\frac{5}{2}$ Set the denominator = 0

$2x + 5 = 0$

$2x = -5$

$x = -\frac{5}{2}$

8. Classify the functions below as exponential growth or decay (in other words, is the function increasing or decreasing):

$f(x) = (\frac{2}{5})^x$ decay

$g(x) = (\frac{3}{2})^{-x}$ decay

$h(x) = e^{0.7x}$ growth

a. if $b<1$, Exp is positive function is decreasing

b. if $b>1$, Exp is negative, function is ↘

c. if $b>1$, Exp is positive, function is ↗

8

9. Kyle deposits $2400 into an account that pays interest at a rate of 6.25% compounded weekly.

a. How long before the account reaches $4,000?

b. How much is in the account after 3 years?

TVM Solver

N = mt = $\boxed{}$ = 425.2622807

I% = 6.25

PV = −2400

PMT = 0

FV = 4000

C/Y = 52

P/Y = 52

move cursor to end of line

÷ 52

answer

t = 8.178120783

$\boxed{t \approx 8.2 \text{ yrs.}}$

b. N = 52*3

I% = 6.25

PV = −2400

PMT = 0

FV = $\boxed{}$

P/Y = 52

C/Y = 52

answer

$2894.63

9

10. J.J. deposits \$1500 into an account paying $6\frac{3}{4}\%$ compounded continuously.

a. How long before the balance is \$2500?

b. What is the balance after 2 years?

$A = Pe^{rt}$

$2500 = 1500e^{.0675t}$

$\frac{2500}{1500} = e^{.0675t}$

$\frac{5}{3} = e^{.0675t}$

$\ln(5/3) = \ln e^{.0675t}$

$\ln(5/3) = .0675t$

$\frac{\ln(5/3)}{.0675} = t$ exact answer

$7.567787019 \approx t$

$7.6 \text{ yrs} \approx t$ approx answer

10

11. Given $f(x) = 2.5e^{.02x}$ where x is the time in minutes and f(x) is the number of bacteria in the culture in thousands. Find the number of bacteria in the culture after 2 hrs, rounded to the nearest hundred.

$f(x) = 2.5e^{.02x}$ $x = \#$ minutes

$f(120) = 2.5e^{.02(120)}$ $f(x) =$ 1000's of bacteria

$f(120) = 2.5e^{2.4}$

$= 27.55794095$ thousand bacteria

27,557.94 bacteria

rounded to nearest hundred

27,600 bacteria

11

12. Rewrite in logarithmic form:

a. $10^x = 2.4$

b. $5^{x+1} = 3$

c. $e^{x^2} = 14$

a. $\log(2.4) = x$

b. $\log_5(3) = x+1$

c. $\ln(14) = x^2$

12

13. $y^2 + 3x = 5$ Does this equation describe a function? Why, or why not?

$y^2 + 3x = 5$

$y^2 = -3x + 5$

$y = \pm\sqrt{-3x+5}$

two values of y, for each x value

No, not a function

13

14. Solve for x: $\log_7(\log(\ln x)) = 1$

$$\log_7(\log(\ln x)) = 1$$

$$\log_7(\text{MESS}) = 1$$

$$7^1 = \text{MESS}$$

$$7 = \log(\ln x)$$

$$7 = \log(\text{mess})$$

$$10^7 = \text{mess}$$

$$10^7 = \ln x$$

$$\boxed{e^{10^7} = x}$$

14

15. Given $\log(x) = 8$ and $\log(y) = 12$, evaluate: $\log(1000x^2y)$

$\log(1000x^2y) =$

$\log 1000 + \log x^2 + \log y =$

$1000 = 10^3$

$\log 10^3 + \log x^2 + \log y =$

$3 \log 10 + 2 \cdot \log x + \log y =$

$3(1) + 2(8) + 12 =$

$3 + 16 + 12 =$

$19 + 12 = \boxed{31}$

15

16. Solve for x: $5 \cdot 3^{2x-1} = 135$

$$5 \cdot 3^{2x-1} = 135$$

$$3^{2x-1} = \frac{135}{5}$$

$$3^{2x-1} = 27$$

$$3^{2x-1} = 3^{3}$$

$b^x = b^y \Rightarrow x = y$

$$2x-1 = 3$$

$$2x = 4$$

$$\boxed{x = 2}$$

16

17. Solve for x: $\log_4 (x^2 - 9) = 2$

$$\log_4 (x^2 - 9) = 2$$
$$4^2 = x^2 - 9$$
$$16 + 9 = x^2$$
$$25 = x^2$$
$$\boxed{\pm 5 = x}$$

remember to check that arguement is positive when you substitute in ans

17

18. Solve for x: $\log_2 (x+1) = 4 - \log_2 (x-5)$

$$\log_2 (x+1) = 4 - \log_2 (x-5)$$

$$\log_2 (x+1) + \log_2 (x-5) = 4$$

$$\log_2 (x+1)(x-5) = 4$$

$$2^4 = (x+1)(x-5)$$

$$16 = x^2 - 4x - 5$$

$$0 = x^2 - 4x - 21$$

$$0 = (x-7)(x+3)$$

$$x - 7 = 0 \qquad , \qquad x + 3 = 0$$

$$\boxed{x = 7} \qquad\qquad \cancel{x = -3}$$

extraneous root

18

19. If planted with 100 trees, each tree produces \$50 per year in produce. Due to overcrowding, for each additional tree planted the yield drops 50 cents. How many trees should be planted to maximize the revenue?

If the cost to care for the trees (fertilizer and water) run \$15 per year per tree, how many trees should be planted to maximize profit?

(x, p) $x = \#$ trees

$(100, 50)$ $p = \$$ produced/tree

$(102, 49)$

Stat, Edit

L_1	L_2
100	50
102	49

Stat → Calc

4: LinReg(ax+b) L_1, L_2, Y_1 enter

$y = ax + b$

$a = -0.5$

$b = 100$

$p = -.5x + 100$

$R = xp$

$R = x(-0.5x + 100)$

$R = -0.5x^2 + 100x$

$h = \frac{-b}{2a} = \frac{-(100)}{2(-0.5)} = 100$

answer: $\boxed{100 \text{ trees}}$

$C = 15x$

$P = R - C$

$P = -0.5x^2 + 100x - (15x)$

$P = -0.5x^2 + 85x$

$h = \frac{-b}{2a} = \frac{-85}{2(-0.5)}$

$h = 85$

answer: $\boxed{85 \text{ trees}}$

19

20. Find the following for $f(x) = \frac{x^2 - 3x - 4}{2x^2 + 5x + 3}$, if they exist.

a. vertical asymptote(s): $x = -3/2$

b. horizontal asymptote(s): $y = 1/2$

c. hole(s) in the graph: $x = -1$

d. x-intercept(s): $(4, 0)$ set numerator $= 0$

e. y-intercept(s): $(0, -4/3)$ let $x = 0$

$$f(x) = \frac{(x-4)\cancel{(x+1)}}{(2x+3)\cancel{(x+1)}}$$

$2x + 3 = 0$

$2x = -3$

$x = -\frac{3}{2}$

20

21. How long does it take for an account earning $10\frac{3}{4}\%$ compounded quarterly to double?

TVM Solver:

$N = mt = 4t = \boxed{}$

$I\% = 10.75$

$PV = -100$ pick any number ie: \$100

$PMT = 0$

$FV = 200$ ⇐ value doubles

$P/Y = 4$

$C/Y = 4$

Move cursor to N + Solve

$N = 26.13656474 = 4t$

$\div 4$ $6.534141185 = t$

$t \approx 6.5$ years

22. Find the present value of an account compounded continuously at $5\frac{1}{4}\%$ if in ten years it grows to \$8000.

$A = Pe^{rt}$ $P = ?$

$r = 0.0525$

$t = 10$

$A = 8{,}000$

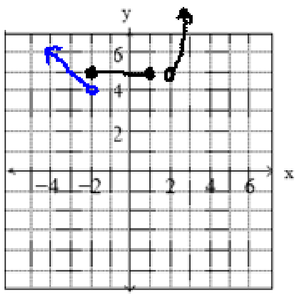
$8000 = Pe^{0.0525(10)}$

$\frac{8000}{e^{.525}} = P$

$\boxed{P \approx \$4{,}732.44}$

23. Graph the piecewise defined function

$$f(x) = \begin{cases} |\, x - 2 \,|, & \text{if } x < -2 \\ 5, & \text{if } -2 \le x \le 1 \\ x^2 + 1 & x > 2 \end{cases}$$



23

24. A machine purchased for \$1800 has a useful life of 6 years and a scrap value of \$500. Assuming a straight line depreciation, find its value at 4 years old.

t = age in years

V = value in dollars

(t, V)

$(0, 1800)$

$(6, 500)$

$$V = -\frac{650}{3}t + 1800$$

$$V(4) = \boxed{\$933.33}$$

24

25. Solve: $25^{3x} = 125^{x-4}$

$$25^{3x} = 125^{x-4}$$

$$(5^2)^{3x} = (5^3)^{x-4}$$

$$5^{6x} = 5^{3x-12}$$

$$6x = 3x - 12$$

$$3x = -12$$

$$\boxed{x = -4}$$

25

26. Find the derivative of $f(x)$ given that
$f(x+h) - f(x) = \frac{3}{2x+2h-5} - \frac{3}{2x-5}$.

$$f(x+h) - f(x) = \frac{3(2x-5) - 3(2x+2h-5)}{(2x+2h-5)(2x-5)}$$

$$f(x+h) - f(x) = \frac{6x-15-6x-6h+15}{(2x+2h-5)(2x-5)} = \frac{-6h}{(2x+2h-5)(2x-5)}$$

$$\frac{f(x+h)-f(x)}{h} = \frac{1}{\cancel{h}} \cdot \frac{-6\cancel{h}}{(2x+2h-5)(2x-5)} = \frac{-6}{(2x+2h-5)(2x-5)}$$

$$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = \frac{-6}{(2x-5)(2x-5)} = \frac{-6}{(2x-5)^2}$$

26

27. Find the derivative using the limit definition of the derivative for

a. $f(x) = \sqrt{4-3x}$.

b. $g(x) = x^2 + 5x - 10$

c. $h(x) = \dfrac{x}{x+2}$

a. $f(x) = \sqrt{4-3x}$

$f(x+h) = \sqrt{4-3(x+h)}$

$f(x+h) = \sqrt{4-3x-3h}$

$f(x+h) - f(x) = \sqrt{4-3x-3h} - \sqrt{4-3x}$

$$\frac{f(x+h)-f(x)}{h} = \frac{\sqrt{4-3x-3h}-\sqrt{4-3x}}{h} \cdot \frac{\sqrt{4-3x-3h}+\sqrt{4-3x}}{\sqrt{4-3x-3h}+\sqrt{4-3x}}$$

$$\frac{f(x+h)-f(x)}{h} = \frac{\cancel{4}-\cancel{3x}-3h-(\cancel{4}-\cancel{3x})}{h\left[\sqrt{4-3x-3h}+\sqrt{4-3x}\right]}$$

$$= \frac{-3\cancel{h}}{\cancel{h}\left[\sqrt{4-3x-3h}+\sqrt{4-3x}\right]} = \frac{-3}{\sqrt{4-3x-3h}+\sqrt{4-3x}}$$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \frac{-3}{\sqrt{4-3x}+\sqrt{4-3x}} = \boxed{\frac{-3}{2\sqrt{4-3x}}}$$

b. $g(x) = x^2+5x-10$

$g(x+h) = (x+h)^2 + 5(x+h) - 10 = x^2+2xh+h^2+5x+5h-10$

$g(x+h) - g(x) = \cancel{x^2}+2xh+h^2+\cancel{5x}+5h-\cancel{10}-(\cancel{x^2}+\cancel{5x}-\cancel{10})$

$= 2xh + h^2 + 5h$

$$\frac{g(x+h)-g(x)}{h} = 2x+h+5$$

$$\lim_{h\to 0} \frac{g(x+h)-g(x)}{h} = \boxed{2x+5}$$

c. $H(x) = \frac{x}{x+2}$

$$H(x+h) = \frac{x+h}{x+h+2}$$

$$H(x+h) - H(x) = \frac{x+h}{x+h+2} - \frac{x}{x+2}$$

$$= \frac{(x+h)(x+2) - x(x+h+2)}{(x+h+2)(x+2)}$$

$$= \frac{\cancel{x^2} + \cancel{2x} + \cancel{hx} + 2h - \cancel{x^2} - \cancel{hx} - \cancel{2x}}{(x+h+2)(x+2)}$$

$$= \frac{2h}{(x+h+2)(x+2)}$$

$$\frac{H(x+h) - H(x)}{h} = \frac{2}{(x+h+2)(x+2)}$$

$$\lim_{h \to 0} \frac{H(x+h) - H(x)}{h} = \frac{2}{(x+2)(x+2)} = \boxed{\frac{2}{(x+2)^2}}$$

28

28. An object moves along the y- axis as described by $y = x^2 + 2x$, where x is the time in seconds, and y is measured in feet.

a. Find the average velocity when x changes from 2 seconds to 5 seconds.

b. Find the instantaneous velocity at x=2 seconds.

a. $f(x) = x^2 + 2x$

$f(2) = 2^2 + 2(2) = 8$ $\quad$ $f(5) = 5^2 + 2(5) = 25 + 10 = 35$

$(2, 8)$ $\quad$ $(5, 35)$

$m = \frac{35-8}{5-2} = \frac{27}{3} = 9$ $\quad$ $\boxed{9 \text{ ft/sec}}$

b. $f(x) = x^2 + 2x$

$f(2) = 8$

$f(2+h) = (2+h)^2 + 2(2+h) = 4 + 4h + h^2 + 4 + 2h$

$= 8 + 6h + h^2$

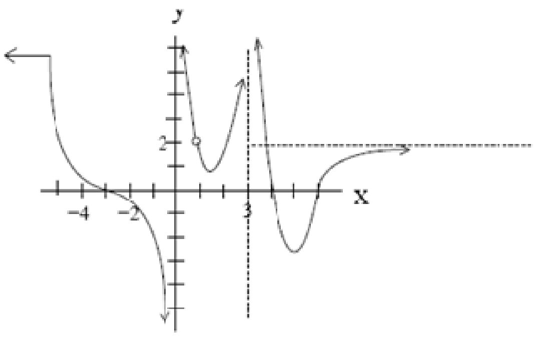
$f(2+h) - f(2) = 8 + 6h + h^2 - 8 = 6h + h^2$

$\frac{f(2+h) - f(2)}{h} = 6 + h$

$\lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = \boxed{6}$ $\quad$ $\boxed{6 \text{ ft/sec}}$

29

29. Refer to the graph of $f(x)$ below. List the values of x for which $f'(x)$ does not exist .



at $x=-5$, corner

at $x=0$, vertical asymptote

at $x=3$, vertical asymptote

at $x=1$, hole in the graph

30

30. The profit in dollars from the sale of x Wii © consoles is given by $P(x) = 200x - 0.01x^2 - 3000$.

a. Find the average change in profit if production changes from 7000 units to 8000 units.

b. Find $P'(x)$.

c. Find the instantaneous rate of change of profit at 7000 units.

d. As the plant manager, would you keep production the same (7000/yr), increase, or decrease production?

$P(x) = 200x - 0.01x^2 - 3000$

a) $P(7{,}000) = 907{,}000$ $\quad m = \frac{957000 - 907000}{8000 - 7000}$

$P(8000) = 957{,}000$ $\quad m = \frac{50{,}000}{1{,}000} = 50$

$50/unit

b) $P'(x) = 200 - 0.02x$

c) $P'(7{,}000) = 200 - 0.02(7{,}000) = 60$

$60/unit

d) increase because derivative is positive (rate of change of profit is increasing)

31

31. How can the derivative be used to find the maximum and minimum?

find where $f' = 0$

32

32. Which of the following functions represent $1-1$ functions?

a. $y = x^2 - 10x + 24$ not 1-1

b. $y = |2x - 5|$ not 1-1

c. $y = \ln(x-5) + 1$ Yes 1-1

33

d. $y = \sqrt[3]{x-1}$ Yes, 1-1

e. $y = e^{x-2}$ Yes, 1-1

34

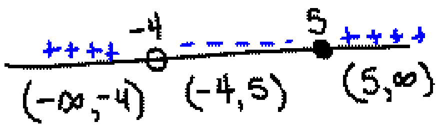
33. Solve the following inequality using a sign chart.

$$\frac{2x-10}{x+4} \geq 0$$

$2x - 10 = 0$ $\quad$ $x + 4 \neq 0$

$2x = 10$ $\quad$ $x \neq -4$

$x = 5$



$x=$	-6	0	6
$2x-10$	−	−	+
$x+4$	−	+	+
Quotient	+	−	+

Where is function positive OR zero

$(-\infty, -4) \cup [5, \infty)$

35

34. Find

$$\lim_{x\to -3} \frac{2x|x+3|}{x+3}$$

$$\lim_{x\to -3^+} \frac{2x|x+3|}{x+3} = -6$$

plug in -2.99 $\quad \dfrac{2(-2.99)|-2.99+3|}{-2.99+3} = -5.98$

plug in -2.999 $\quad \dfrac{2(-2.999)|-2.999+3|}{-2.999+3} = -5.998$

$$\lim_{x\to -3^-} \frac{2x|x+3|}{x+3}$$

plug in -3.001 $\quad \dfrac{2(-3.001)|-3.001+3|}{-3.001+3} = 6$

$$\lim_{x\to -3} \frac{2x|x+3|}{x+3} \text{ dne}$$

36

35. Find

$$\lim_{x \to -2} \sqrt{3x^2 + 5x + 2} =$$

$$\sqrt{3(-2)^2 + 5(-2) + 2} =$$

$$\sqrt{12 - 10 + 2} =$$

$$\sqrt{4} =$$

$$2$$

37

36. Find

$$\lim_{x \to 4} (2x^2 - 5x + 1)$$

$2(4)^2 - 5(4) + 1 =$

$32 - 20 + 1 =$

$12 + 1 = 13$

37. Find $\lim_{x \to 2+} f(x)$ *AND* $\lim_{x \to 2-} f(x)$ when

$$f(x) = \begin{cases} |x-3|, & x < 2 \\ x^2 - 1, & x \geq 2 \end{cases}$$

$$\lim_{x \to 2^+} f(x) = 3$$

$$\lim_{x \to 2^-} f(x) = 1$$

39

38. Find: $\lim_{x \to \infty} \frac{3x - 4}{x - 1}$ = 3

39. Find: $\lim_{x \to -\infty}(6 - 2x - x^3)$ = ∞

41

40. Find the inverse, y_2, of $y_1 = \ln(2x - 30)$.

State the domain and range of y_1 and y_2.

$$y_1 = \ln(2x - 30)$$
$$x = \ln(2 \cdot y_2 - 30)$$
$$e^x = 2 \cdot y_2 - 30$$
$$e^x + 30 = 2 \cdot y_2$$
$$\frac{1}{2}e^x + 15 = y_2$$

	y_1	y_2
domain	$x > 15$	$\mathbb{R}$
range	$\mathbb{R}$	$y > 15$

42

41. Find $\lim_{x \to 4} \frac{5x^2 - 28x + 32}{x^2 - 16}$ $= \frac{0}{0}$

indeterminant form

$$\lim_{x \to 4} \frac{(5x-8)\cancel{(x-4)}}{\cancel{(x-4)}(x+4)} = \lim_{x \to 4} \frac{5x-8}{x+4} = \frac{20-8}{4+4} = \frac{12}{8}$$

$$= \boxed{\frac{3}{2}}$$

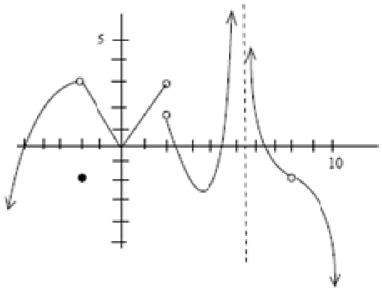
43

42. State the best reason why $f(x)$ shown below is not continuous at each point of discontinuity:

i. $f(a)$ is not defined. at $x=8$

ii. $\lim_{x \to a} f(x)$ is not defined. at $x=2$, $x=5.5$

iii. $\lim_{x \to a} f(x) \neq f(a)$ at $x=-2$



44

43. The revenue from the sale of Valentine Candy is given by $R(x) = 40x - 0.25x^2$ over the interval $0 \leq x \leq 150$.

a. What is the change in revenue if sales change from 50 boxes of Valentine candy to 80 boxes?

b. Find the average change in revenue for part (a).

a. $R(50) = 1375$

$R(80) = 1600$ difference $225.

b. $m = \frac{1600-1375}{80-50} = \frac{225}{30} = \frac{75}{10} = 7.5$

$7.50/box

45

44. The derivative does not exist under what conditions?

a. Corner or cusp

b. Vertical asymptote

c. hole or skip

46

45. a. Find the maximum revenue for *Hot Air Balloon Adventures* where x represents the number of rides provided weekly and $R(x)$ is measured in dollars. They have a price-demand equation of $p(x) = 100 - x$.

b. How many rides must be scheduled to maximize the revenue?

c. If the fixed costs for *Hot Air Balloon Adventures* is \$900/wk and the total costs to provide 10 rides is \$1150, find the break-even point(s).

$p = 100 - x$

$R = xp = x(100-x)$

$R = 100x - x^2$

$R' = 100 - 2x = 0$

$100 = 2x$

$50 = x$

$R(50) = 100(50) - 50^2$

$R(50) = 2500$

max revenue $\boxed{\$2500}$

b. $\boxed{50 \text{ rides}}$

c. $f = 900$

$C = 1150$

$x = 10$

$V =$

$C = f + v \cdot x$

$1150 = 900 + 10v$

$250 = 10v$

$25 = v$

$C = 900 + 25x$

$R = 100x - x^2$

$R = C$

$100x - x^2 = 900 + 25x$

$0 = x^2 - 75x + 900$

$0 = (x-15)(x-60)$

$x = 15, \quad x = 60$

Break-even Pts

$(15, 1275)\ (60, 2400)$

47

46. The derivative of $f(x) = 4x^2 + 5x - 10$ using the power rule, and constant rule is:

$f(x) = 4x^2 + 5x - 10$

$\boxed{f'(x) = 8x + 5}$

48

47. Given $f(x) = 2x^2 - x + 1$

a. find the slope of the tangent line at $x = 4$.

b. write the equation of the tangent line.

a. $f'(x) = 4x - 1$

$f'(4) = 4(4) - 1 = \boxed{15}$

b. $f(4) = 2(4)^2 - 4 + 1$

$= 32 - 4 + 1$

$= 29$

$(4, 29)$

$m = 15$

$(4, 29)$

$y - 29 = 15(x - 4)$

$y - 29 = 15x - 60$

$\boxed{y = 15x - 31}$

49

48. Find the derivative, y', if

a. $y = x^{-4} + x^{\frac{3}{2}} - e^3$.

b. $y = \dfrac{1}{\sqrt[3]{x^2}}$.

c. $y = 4.2x^{-2} - \dfrac{0.5}{\sqrt[4]{x}} + 2$.

d. $y = x^2 - 1.5x - 10\sqrt{x}$.

e. $y = \dfrac{x^5 - 5x^3 - 2}{x^2}$.

a. $y = x^{-4} + x^{3/2} - e^3$

$y' = -4x^{-3} + \frac{3}{2}x^{1/2} + 0$

$y' = \frac{-4}{x^3} + \frac{3}{2}x^{1/2}$

b. $y = \frac{1}{\sqrt[3]{x^2}} = x^{-2/3}$

$y' = -\frac{2}{3}x^{-5/3} = \frac{-2}{3x^{5/3}}$

c. $y = 4.2x^{-2} - \frac{0.5}{\sqrt[4]{x}} + 2$

$y = 4.2x^{-2} - 0.5x^{-1/4} + 2$

$y' = -8.4x^{-3} + \frac{1}{2}\left(\frac{1}{4}\right)x^{-5/4} + 0$

$y' = \frac{-8.4}{x^3} + \frac{1}{8x^{5/4}}$

d. $y = x^2 - 1.5x - 10\sqrt{x}$

$y = x^2 - 1.5x - 10x^{1/2}$

$y' = 2x - 1.5 - 5x^{-1/2}$

$y' = 2x - 1.5 - \frac{5}{x^{1/2}}$

e. $y = \frac{x^5 - 5x^3 - 2}{x}$

$y = \frac{x^5}{x} - \frac{5x^3}{x} - \frac{2}{x}$

$y = x^4 - 5x^2 - 2x^{-1}$

$y' = 4x^3 - 10x + 2x^{-2}$

$y' = 4x^3 - 10x - \frac{2}{x^2}$

50

49. The price-demand function and the cost function for the production of air-conditioning units is $x = 2000 - 0.25p$ and $C(x) = 60,000 + 200x$.

a. Find the average cost of making 100 units.

b. Find the marginal cost of making 100 units.

c. Find the marginal average cost when $x = 100$.

d. Find the revenue when 100 units are made and sold.

$x = 2,000 - \frac{1}{4}p$

$4x = 8,000 - p$

$p = 8,000 - 4x$

$R = xp = x(8000 - 4x)$

$R = 8000x - 4x^2$

a. $C = 60,000 + 200x$

$\bar{C} = \frac{C}{x} = \frac{60,000}{x} + 200$

$\bar{C}(100) = \frac{60,000}{100} + 200 = \boxed{\$800}$

b. $C = 60,000 + 200x$

$MC = C' = 0 + 200$

$MC = \$200$

c. $AC = 60000x^{-1} + 200$

$MAC = -60,000x^{-2} + 0 = \frac{-60,000}{x^2}$

$MAC(100) = \frac{-60,000}{(100^2)} = \boxed{-6}$

d. $R = x(8,000 - 4x)$

$R(100) = 100(8,000 - 400) = \boxed{\$760,000}$

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e. Find the average revenue when 100 units are made and sold.

f. Find the revenue of making and selling 25 units.

g. Find the approximate revenue from the 25^{th} unit.

h. Find the marginal average revenue function.

e. $R = 8000x - 4x^2$

$AR = \bar{R} = 8000 - 4x$

$\bar{R}(100) = 8000 - 400$

$= \boxed{\$7,600}$

f. $R = 8000x - 4x^2$

$R(25) = 8000(25) - 4(25)^2$

$R(25) = \boxed{\$197,500}$

g. $R = 8000x - 4x^2$

$R' = 8000 - 8x$

$R'(24) = 8000 - 8(24)$

$= \boxed{\$7808}$

h. $R = 8000x - 4x^2$

$\frac{R}{x} = \bar{R} = 8000 - 4x$

$MAR = \boxed{-4}$

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i. What is the profit from making and selling 100 units?

j. What is the marginal profit function.

k. Find the marginal average cost function.

l. How many should they make and sell to maximize revenue?

i. $P = R - C$

$P = 8000x - 4x^2 - [60{,}000 + 200x]$

$P = 8000x - 4x^2 - 60{,}000 - 200x$

$P = -4x^2 + 7800x - 60{,}000$

$P(100) = -4(100)^2 + 7800(100) - 60{,}000$

$P(100) = \boxed{\$680{,}000}$

j. $P = 8000x - 4x^2 - 60{,}000 - 200x$

$P = 7800x - 4x^2 - 60{,}000$

$MP = \boxed{7800 - 8x}$

k. $MAC = \dfrac{-60{,}000}{x^2}$

l. $R = 8000x - 4x^2$

$R' = 8000 - 8x = 0$

$8000 = 8x$

$\boxed{1{,}000 = x}$

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m. How many should they make and sell to maximize profit?

$$P = -4x^2 + 7800x - 60{,}000$$

$$P' = -8x + 7800 = 0$$

$$7800 = 8x$$

$$\boxed{975 = x}$$

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