Review for EXAM # 3

MATH 142 Drost-Fall 2007

5.4 Curve Sketching Techniques

1. Graph and label relative extrema, inflection points, and asymptotes for the function: $f(x) = \dfrac{3x+1}{5x-2}$
2. Find the relative extrema, inflection points, and asymptotes for the function $f(x) = x \cdot e^{-\frac{1}{2}x}$.
3. The Digital Pet Company has determined that its daily cost in dollars, for producing x virtual pets is given by $C(x) = 1500 + 3x + \dfrac{x^2}{15}$, $\quad 0 \le x \le 200$. Find the average cost function.
4. How many virtual pets should they make to minimize the average costs?
5. Find where the function is concave up or down for

   $$f(x) = -3x^3 + 5x^2 - 2x + 4$$

6. Find where $g(x)$ is increasing/decreasing, concave up/down, relative extrema, and all asymptotes.

   $$g(x) = 2x + \frac{50}{x}$$

7. Sketch the graph described below:

   domain $\Re$, except $x = -2, x = 3$

   $f(0) = 0$

   range $\Re$

   horizontal asymptote: $y = 1$

   $f' < 0 \quad (3, \infty)$

   $f' > 0 \quad (-\infty, -2), (-2, 3)$

   $f" > 0 \quad (-\infty, -2), (0, 3), (3, \infty)$

   $f" < 0 \quad (-2, 0)$

5.5 Absolute Extrema

8. Determine the absolute extrema:

   $f(x) = \dfrac{x}{x-2}$ on the interval $[3, 5]$.

9. Find the absolute minimum value on the interval $[0, \infty)$ for the function $f(x) = (x+4)(x-2)^2$.
10. Find the absolute maximum value on the interval $(0, \infty)$ for the function $f(x) = \dfrac{x^4}{e^x}$.
11. Given $f(x) = 2x^3 - 3x^2 - 72x - 150$, find the absolute extrema on the interval $[-5, 7.5]$
12. Given $f(x) = \dfrac{x-1}{x^2}$, find the absolute extrema on the interval $(-\infty, \infty)$

5.6 Optimization

13. From past records, the owner of the Sleep Cheap Motel has determined that when \$x per day is charged to rent a room the daily profit, $P(x)$, is given by

    $$P(x) = -x^2 + 92x - 180, \quad 40 \le x \le 60$$

    What should the owner charge to maximize profits?

14. Find the dimensions of a garden, x feet by y feet, where $10 \le x \le 100$, so that the garden will have 1440 sq. ft of area, and be surrounded by a walkway that is 8ft wide on the north and south sides, and 5ft wide on the east and west sides, so that the Total Area (of garden and walkways) is a mimimum.
15. The concentration of a certain medication in a patient's bloodstream can be given by $C(t) = \dfrac{5.3t}{t^2+4t+5}, 0 \le t \le 8$ where $C(t)$ is in milligrams per cubic centimeter and t is the number of hours after the medication has been administered.

    a. How many hours after the medication has been administered is the concentration at a maximum?

    b. What is the maximum concentration?

16. Find two numbers whose sum is 20 for which the sum of the squares is a minimum.
17. Find the dimensions of a closed box with a square base which has a volume of 27000 cubic inches with minimum surface area.
18. The local travel agency offers one week in Hawaii, for \$1120 if 20 sign up, and discounts of \$20 per person for each additional person over the minimum of 15. If the 747 holds 300, what price should they charge to maximize revenue?

6.1 Antiderivatives and Indefinite Integrals

19. $\displaystyle\int \sqrt[4]{x^5}\,dx$
20. $\displaystyle\int (\sqrt{x} - \sqrt[3]{x})\,dx$
21. $\displaystyle\int \frac{x^3 + x^5}{x^4}\,dx$
22. $\displaystyle\int (e^x + \pi)\,dx$
23. Find the cost function if the marginal cost in dollars is given by $28x - 10e^x$ where x is the number of items sold and there are fixed costs of \$50.

24. The population of Smallsville, OH is increasing at the rate $2400e^{.02t}$ where t is the number of years since 1950, when the population was 30,000. Find the population in 1985 according to this model.

6.2 Integration by Substitution or "Fix-It" Method

25. $\int 6(x+1)(2x^2+4x-5)^{\frac{3}{2}}\ dx$

26. $\int 5x(3x^2-10)^{\frac{1}{2}}\ dx$

27. $\int \frac{x}{\sqrt[3]{x^2-1}}\ dx$

28. $\int \frac{x^3}{\sqrt{x^4-4}}\ dx$

29. $\int_{-6}^{2} \sqrt[3]{2-x}\ dx$

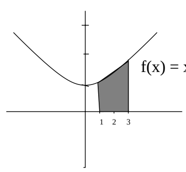
30. $\int \frac{1}{x^2}e^{\frac{-1}{x}}\ dx$

31. $\int x\sqrt{x+1}\ dx$

32. $\int x(x+6)^8\ dx$

6.4 The Definite Integral

33. Write the definite integral to represent the shaded area:

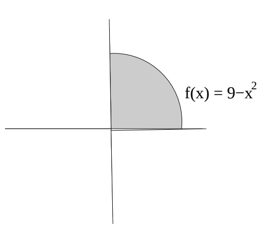


34. Find the approximate area under the curve $f(x) = \frac{1}{4}x^2 + 2$ over the interval $[-2, 4]$ using 3 rectangles.

35. Write a definite integral to represent the shaded area under the curve and above the x-axis, over the interval $[0, 5]$

$$f(x) = \begin{cases} 6-x, & if \quad x \le 3 \\ x, & if \quad x > 3 \end{cases}$$

36. Write a definite integral to represent the shaded area.



37. Sketch and shade the region given by the definite integral $\int_0^{13} 4\sqrt[3]{2x+2}\, dx$

6.5 The Fundamental Theorem of Calculus

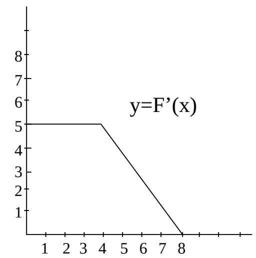
38. $\int_{-2}^{2} (4-x^2)\ dx$

39. $\int_1^5 e^{2x}\ dx$

40. $\int_0^1 xe^{x^2+1}\ dx$

41. $\int_0^1 \frac{1}{\sqrt{3x+1}}\ dx$

42. Refer to the figure below of $F'(x)$. If F(0)=2, what is F(4)? What is F(8)?



43. Suppose oil is being extracted at a rate $.1e^{.5t}$ where t is measured in years and P(t) in millions of barrels of oil. At this rate, how much oil will be extracted the 5th year?

44. The ScandiTrac Company determines that their marginal profit function for producing and selling a new economy model of cross-country ski machine at a mall is given by

$MP(x) = P'(x) = 0.3x^2 + 0.2x, \quad 0 \le x \le 30$

where x is the number of machines produced and sold and $P'(x)$ is the marginal profit function measured in dollars per ski machine.

a. Knowing that \$704 profit is made when 20 ski machines are sold, find the profit function $P(x)$.

b. Evaluate $\int_{10}^{20} P'(x)dx$ and interpret.

45. Suppose my salary over the last forty years is approximated by the function $S = 4188x^{0.7}$.

a. What was my average salary over the first five years?

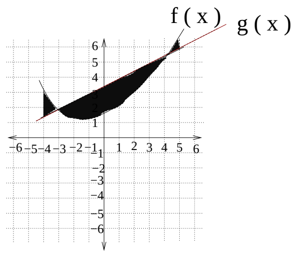
b. What was my average salary over the last five years?

7.1 Area Between Curves

46. Find the area between $y_1 = x^2 + 2$ and $y_2 = 0$ on the interval $[0, 3]$.

47. Find the area between $y_1 = \frac{1}{x}$ and $y_2 = 0$ on the interval $[1, e]$.

48. Write the definite integral to describe the shaded area above.



49. Find the area between $y_1 = x^3 - 6x^2 + 9x$ and $y_2 = x$.

50. Using data from the US Census Bureu, an economist produced the following Lorenz curve for the distribution of income in 1972: $f(x) = 0.5x + 0.5x^2$. Find the Gini index of income concentration.

51. The data in the table describes the distribution of wealth in a country.

x	0	0.20	0.40	0.60	0.80	1
y	0	.2	.4	.8	.4	1

a. Use quadratic regression to find the equation of a Lorenz curve for the data.

b. Approximate the Gini index of the income concentration.

7.2 Applications in Business and Economics

52. Find the consumers' surplus at a price level of $\overline{p} = \$150$ for the price-demand equation $p = D(x) = 400 - 0.05x$.

53. Find the producers' surplus at the equilibruium price if $p = D(x) = 25 - 0.004x^2$ and $p = S(x) = 5 + 0.004x^2$

54. In the figure below: $D(x) = 25 - .001x^2$ and $S(x) = 5 + 0.1x$

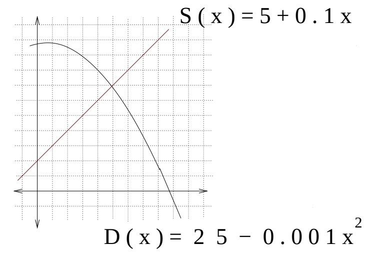
a. Shade the consumers' surplus at a price level of \$16.90.

b. Find the equilibrium point.

c. Find the equilibrium price.

d. Find the equilibrium quantity.

e. Find the producers' surplus at the equilibrium point.



Additional Examples

55. $\displaystyle\int \frac{1}{2x+3}\ dx$

56. $\displaystyle\int \frac{6x^2-6}{x^3-3x+4}\ dx$

57. $\displaystyle\int \frac{1}{x\ln x}\ dx$

58. $\displaystyle\int \frac{e^{2x}+e^{-2x}}{e^{2x}-e^{-2x}}\ dx$

59. $\displaystyle\int 4x \cdot 5^{x^2+1} dx$

60. $\displaystyle\int (x + \frac{1}{x}) dx$

Answer Key for Exam 3 Review

5.4 Curve Sketching

1. rel extrema: none

   inflection pts: none

   v.a.: $x = \frac{2}{5}$

   h.a.: $y = \frac{3}{5}$

2. rel max at $x = 2$, point of inflection at $x = 4$, h.a. $y = 0$

3. $AC = \frac{1500}{x} + 3 + \frac{1}{15}x$

4. 150 virtual pets

5. $\cup : (-\infty, \frac{5}{9}, \cap : (\frac{5}{9}, \infty)$

6. $\nearrow (-\infty, -5), (5, \infty)$

   $\searrow (-5, 0), (0, 5)$

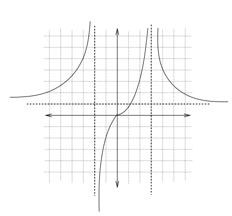
   $\cap(-\infty, 0), \cup(0, \infty)$

   rel max at $x = -5$

   rel min at $x = 5$

   oblique asym: $y = 2x$; vert asym: $x = 0$

7.



5.5 **Absolute Extrema**

8. absolute max =3; absolute min = $1\frac{2}{3}$

9. 0

10. $\dfrac{256}{e^4}$

11. abs min -358, abs max -15

12. no min, max 0.25

5.6 **Optimization**

13. \$46 /day

14. 30 feet by 48 feet

15. $t = 2.236$ hrs; b. $.63 mg/cm^3$

16. x=10, y=10

17. 30" by 30" by 30"

18. $p = \$760$

6.1 **Antiderivatives**

19. $\frac{4}{9}x^{\frac{9}{4}} + C$

20. $\frac{2}{3}x^{\frac{3}{2}} - \frac{3}{4}x^{\frac{4}{3}} + C$

21. $\ln \mid x \mid + \frac{1}{2}x^2 + C$

22. $e^x + \pi x + C$

23. $C(x) = 14x^2 - 10e^x + 60$

24. approx 151,650 people

6.2 **Integration by Substitution**

25. $\frac{3}{5}(2x^2 + 4x - 5)^{\frac{5}{2}} + C$

26. $\frac{5}{9}(3x^2 - 10)^{\frac{3}{2}} + C$

27. $\frac{3}{4}(x^2 - 1)^{\frac{2}{3}} + C$

28. $\frac{1}{2}(x^4 - 4)^{\frac{1}{2}} + C$

29. 12

30. $e^{-\frac{1}{x}} + C$

31. $\frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}} + C$

32. $\frac{1}{10}(x+6)^{10} - \frac{2}{3}(x+6)^9 + C$

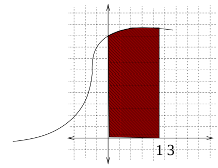
6.4 **The Definite Integral**

33. $\int_1^3 (x^2 + 1)\ dx$

34. $L_3 = 16, R_3 = 22$

35. $A = \int_0^3 (6 - x)dx + \int_3^5 x dx$

36. $\int_0^3 (9 - x^2)\ dx$



37.

6.5 **The Fundamental Theorem of Calculus**

38. $10.\overline{6}$

39. exact: $\frac{1}{2}e^{10} - \frac{1}{2}e^2$
    approx: 11,009.538

40. exact: $\frac{1}{2}e(e-1)$

41. $\frac{2}{3}$

42. 22; 32

43. 960,000 barrels of oil

44a. $P = 0.1x^3 + 0.1x^2 - 136$

44b. \$730; At a production level of 20 machines, the profit from the last 10 machines is \$730.

45. a) \$7600.41

45. b) $\$52,937.20$

7.1 **Area Between Curves**

46. 15

47. 1

48. $\int_{-4}^{-3} [f(x) - g(x)]\ dx + \int_{-3}^{4} [g(x) - f(x)]\ dx + \int_4^5 [f(x) - g(x)]\ dx$

49. 8

50. 0.1667

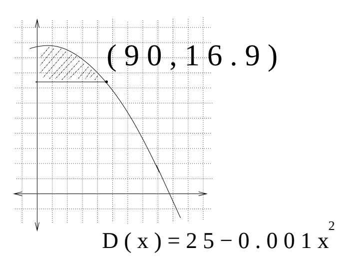
51. $y = -0.179x^2 + 1.036x + 0.014$
    b. 0.055

7.2 **Applications in Business and Economics**

52. $625,000$

53. $333\frac{1}{3}$

54a.



b. $(100, 15)$

c. 15

d. 100

e. 500

Additional Examples

55. $\frac{1}{2} \ln \mid 2x + 3 \mid + C$

56. $2 \ln \mid x^3 - 3x + 4 \mid + C$

57. $\ln \mid \ln \mid x \mid\mid + C$

58. $\frac{1}{2} \ln \mid e^{2x} - e^{-2x} \mid + C$

59. $\dfrac{2 \cdot 5^{x^2+1}}{\ln 5} + C$

60. $\dfrac{x^2}{2} + \ln \mid x \mid + C$