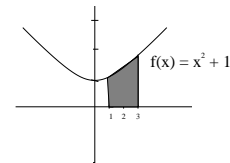


Review for EXAM # 3

MATH 142

Drost-Spring 2010



6.1 Antiderivatives and Indefinite Integrals

- $\int \sqrt[4]{x^5} dx$
- $\int (\sqrt{x} - \sqrt[3]{x}) dx$
- $\int \frac{x^3 + x^5}{x^4} dx$
- $\int (e^x + \pi) dx$
- Find the cost function if the marginal cost in dollars is given by $28x - 10e^x$ where x is the number of items sold and there are fixed costs of \$50.
- The population of Smallsville, OH is increasing at the rate $2400e^{.02t}$ where t is the number of years since 1950, when the population was 30,000. Find the population in 1985 according to this model.
- Find $F(x)$ given $F'(x) = \int (9 - x) \cdot (8 + x^2) dx$, when $F(0) = 5$ and $F'(0) = -4$.

6.2 Integration by Substitution or "Fix-It" Method

- $\int 6(x+1)(2x^2 + 4x - 5)^{\frac{3}{2}} dx$
- $\int 5x(3x^2 - 10)^{\frac{1}{2}} dx$
- $\int \frac{x}{\sqrt[3]{x^2 - 1}} dx$
- $\int_{-6}^2 \sqrt[3]{2-x} dx$
- $\int \frac{1}{x^2} e^{\frac{-1}{x}} dx$
- $\int x\sqrt{x+1} dx$

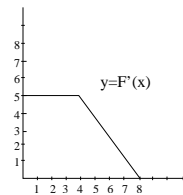
6.4 The Definite Integral

- Write a definite integral to represent the shaded area under the curve and above the x -axis, over the interval $[0, 5]$
$$f(x) = \begin{cases} 6 - x, & \text{if } x \leq 3 \\ x, & \text{if } x > 3 \end{cases}$$
- Find the approximate area under the curve $f(x) = \frac{1}{4}x^2 + 2$ over the interval $[-2, 4]$ using 3 rectangles.

- Write the definite integral to represent the shaded area:

6.5 The Fundamental Theorem of Calculus

- $\int_{-2}^2 (4 - x^2) dx$
- $\int_1^5 e^{2x} dx$
- $\int_0^1 xe^{x^2+1} dx$
- $\int_0^1 \frac{1}{\sqrt{3x+1}} dx$
- Refer to the figure below of $F'(x)$. If $F(0)=2$, what is $F(4)$? What is $F(8)$?



- Suppose oil is being extracted at a rate $.1e^{-.5t}$ where t is measured in years and $P(t)$ in millions of barrels of oil. At this rate, how much oil will be extracted the 5th year?
- The ScandiTrac Company determines that their marginal profit function for producing and selling a new economy model of cross-country ski machine at a mall is given by

$$MP(x) = P'(x) = 0.3x^2 + 0.2x, \quad 0 \leq x \leq 30$$

where x is the number of machines produced and sold and $P'(x)$ is the marginal profit function measured in dollars per ski machine.

a. Knowing that \$704 profit is made when 20 ski machines are sold, find the profit function $P(x)$.

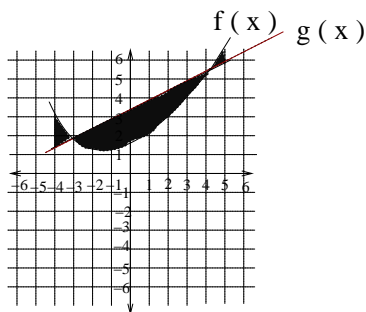
b. Evaluate $\int_{10}^{20} P'(x) dx$ and interpret.

- Suppose my salary over the last forty years is approximated by the function $S = 4188x^{0.7}$.
 - What was my average salary over the first five years?
 - What was my average salary over the last five years?

$$\text{Average Value} = \frac{1}{b-a} \int_a^b f(x) dx$$

7.1 Area Between Curves

25. Find the area between $y_1 = x^2 + 2$ and $y_2 = 0$ on the interval $[0, 3]$.
26. Find the area between $y_1 = \frac{1}{x}$ and $y_2 = 0$ on the interval $[1, e]$.
27. Write a definite integral to describe the shaded area below, where $g(x)$ is linear.



28. Using data from the US Census Bureau, an economist produced the following Lorenz curve for the distribution of income in 1972: $f(x) = 0.5x + 0.5x^2$. Find the Gini index of income concentration.

$$\text{Gini index} = 2 \int_0^1 [x - f(x)] dx$$

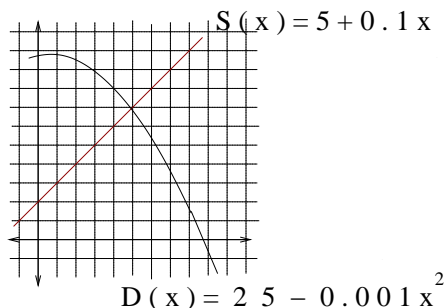
7.2 Applications in Business and Economics

29. Find the consumers' surplus at a price level of $\bar{p} = \$150$ for the price-demand equation

$$p = D(x) = 400 - 0.05x$$

30. Find the producers' surplus at the equilibrium price if $p = D(x) = 25 - 0.004x^2$ and $p = S(x) = 5 + 0.004x^2$

31. In the figure below: $D(x) = 25 - .001x^2$ and $S(x) = 5 + 0.1x$



- a. Shade the consumers' surplus at a price level of \$16.90.

- b. Find the equilibrium point.

- c. Find the equilibrium price.
- d. Find the equilibrium quantity.
- e. Find the producers' surplus at the equilibrium point.

32. Find the equilibrium point given the price-supply equation is $p = S(x) = 5 + .5x^2$ and the price-demand equation is $p = D(x) = -2x + 75$.
33. Find the consumers surplus at the equilibrium point for the previous problem.
34. Find the producers surplus at the equilibrium point, rounded to the nearest cent.

Additional Examples

35. $\int \frac{1}{2x+3} dx$

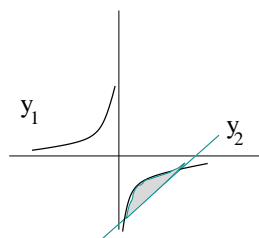
36. $\int \frac{6x^2 - 6}{x^3 - 3x + 4} dx$

37. $\int \frac{1}{x \ln x} dx$

38. $\int (x + \frac{1}{x}) dx$

39. Sammy's Swing Sets INC. manufactures x swing sets per month. The marginal profit each month is given by $P'(x) = 148 - 2x$, on the interval $0 \leq x \leq 74$. They are currently manufacturing 60 swing sets each month, but are planning to increase the production to 70 swing sets monthly. Find the change in the monthly profit.

40. Find the area bounded by $y_1 = \frac{-1}{x}$ and $y_2 = \frac{20}{3}x - \frac{32}{3}$ which is shaded below. Round your answer to two decimal places.



8.1 Functions of Several Variables

41. Evaluate the function

$$f(x, y) = \sqrt{8 - x - y^2}$$

- a. $(2, 1)$,
- b. $(-4, 2)$,
- c. $(-1, 0)$

Find the domain of each of the following:

42. $f(x, y) = \frac{y}{x - 2y}$

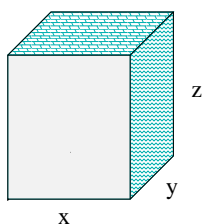
43. $f(x, y) = \sqrt{8 - x + y}$

44. $f(x, y) = 3x^2\sqrt{y}$

45. $f(x, y) = \sqrt[3]{16 + x^2 + y}$

46. A company sells gadgets and widgets. The gadgets sell at $p = 250 - 6x - 4y$ and the widgets sell at $q = 800 - 2x - 5y$, where x = the number of gadgets sold and y = the number widgets sold. Find the revenue function, $R(x, y)$, and the value of $R(5, 6)$.

47. Write a function for the surface area, $S(x, y)$ of a closed rectangular box, in terms of width, x , and length, y , and whose volume is 1000 ft^3 .



8.2 Partial Derivatives: find f_x and f_y for each of the following functions.

48. $f(x, y) = 3x + 4y^2 - 2xy$

49. $f(x, y) = \sqrt{8 - x^2 - y^2}$

50. $f(x, y) = x^4 \cdot e^{2xy}$

51. $f(x, y) = \frac{x}{y^2 - 1}$

52. $f(x, y) = \ln(x^2 + 4y)$

Second Order Partial Derivatives: find the four second-order partial derivatives for each of the following functions.

53. $f(x, y) = 3x^2 + 4y^2 - 2x^2y^3$

54. $f(x, y) = e^{x-2y}$

55. $f(x, y) = \ln(2x + y^2)$

56. $f(x, y) = (5x + 6y^2)^4$

8.3 Maxima and Minima

57. Minimize the surface area in problem number 47.

58. A company sells x logic games at a price $p = 120 - 2x - 3y$ and y super-duper deluxe logic games at a price of $q = 150 + 3x - 5y$.

a. Find the quantity they should sell of each product to maximize revenue.

b. Find the price of each product which maximizes revenue.

Answer Key for Exam 3 Review

6.1 Antiderivatives

1. $\frac{4}{9}x^{\frac{9}{4}} + C$

2. $\frac{2}{3}x^{\frac{3}{2}} - \frac{3}{4}x^{\frac{4}{3}} + C$

3. $\ln|x| + \frac{1}{2}x^2 + C$

4. $e^x + \pi x + C$

5. $C(x) = 14x^2 - 10e^x + 60$

6. approx 151,650 people

7. $F(x) = 36x^2 + \frac{3}{4}x^4 - \frac{4}{3}x^3 - \frac{1}{20}x^5 - 4x + 5$

6.2 Integration by Substitution

8. $\frac{3}{5}(2x^2 + 4x - 5)^{\frac{5}{2}} + C$

9. $\frac{5}{9}(3x^2 - 10)^{\frac{3}{2}} + C$

10. $\frac{3}{4}(x^2 - 1)^{\frac{2}{3}} + C$

11. 12

12. $e^{-\frac{1}{x}} + C$

13. $\frac{2}{5}(x + 1)^{\frac{5}{2}} - \frac{2}{3}(x + 1)^{\frac{3}{2}} + C$

6.4 The Definite Integral

14. $A = \int_0^3 (6 - x) dx + \int_3^5 x dx$

15. $L_3 = 16, R_3 = 22$

16. $\int_1^3 (x^2 + 1) dx$

6.5 The Fundamental Theorem of Calculus

17. $10\sqrt{e}$

18. exact: $\frac{1}{2}e^{10} - \frac{1}{2}e^2$

approx: 11,009.538

19. exact: $\frac{1}{2}e(e - 1)$

20. $\frac{2}{3}$

21. $F(4) = 22; F(8) = 32$

22. 960,000 barrels of oil

23. a. $P = 0.1x^3 + 0.1x^2 - 136$

b. \$730; At a production level of 20 machines, the profit from the last 10 machines is \$730.

24. a) \$7600.41, b) \$52,937.20

7.1 Area Between Curves

25. 15

26. 1

27. $\int_{-4}^{-3} [f(x) - g(x)] dx + \int_{-3}^4 [g(x) - f(x)] dx + \int_4^5 [f(x) - g(x)] dx$

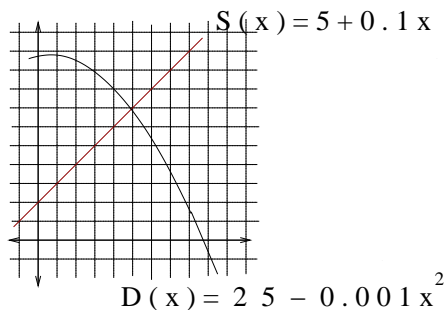
28. 0.1667

7.2 Applications in Business and Economics

29. \$625,000

30. \$333.33

31. a.



b. (100, 15)

c. 15

d. 100

e. 500

32. (10, 55)

33. \$100

34. \$333.33

Additional Examples

35. $\frac{1}{2} \ln |2x + 3| + C$

36. $2 \ln |x^3 - 3x + 4| + C$

37. $\ln |\ln |x|| + C$

38. $\frac{x^2}{2} + \ln |x| + C$

39. \$180.00

40. 4.7586

8.1 Functions of Several Variables

41. a. $\sqrt{5}$, b. $\sqrt{8}$, c. 3

42. $y \neq \frac{1}{2}x$

43. $y \geq x - 8$

44. $y \geq 0$

45. \mathcal{R}

46. $R(x, y) = (250 - 6x - 4y)x + (800 - 2x - 5y)y$
 $R(5, 6) = \$5540$

47. $S_A(x, y) = 2xy + \frac{2000}{y} + \frac{2000}{x}$

8.2 Partial Derivatives

48. $f_x = 3 - 2y$; $f_y = 8y - 2x$

49. $f_x = -x(8 - x^2 - y^2)^{-\frac{1}{2}}$
 $f_y = -y(8 - x^2 - y^2)^{-\frac{1}{2}}$

50. $f_x = 2x^3 e^{2xy}(xy + 2)$, $f_y = 2x^5 e^{2xy}$

51. $f_x = \frac{1}{y^2 - 1}$, $f_y = \frac{-2xy}{(y^2 - 1)^2}$

52. $f_x = \frac{2x}{x^2 + 4y}$, $f_y = \frac{4}{x^2 + 4y}$

53. $f_{xx} = 6 - 4y^3$, $f_{xy} = -12xy^2$, $f_{yx} = -12xy^2$,
 $f_{yy} = 8 - 12x^2y$

54. $f_{xx} = e^{x-2y}$, $f_{xy} = -2e^{x-2y}$, $f_{yx} = -2e^{x-2y}$,
 $f_{yy} = 4e^{x-2y}$

55. $f_{xx} = \frac{-4}{(2x + y^2)^2}$, $f_{xy} = \frac{-4y}{(2x + y^2)^2}$,
 $f_{yx} = \frac{-4y}{(2x + y^2)^2}$, $f_{yy} = \frac{4x - 2y^2}{(2x + y^2)^2}$

56. $f_{xx} = 300(5x + 6y^2)^2$, $f_{xy} = 720y(5x + 6y^2)^2$,
 $f_{yx} = 720y(5x + 6y^2)^2$, $f_{yy} = 48(42y^2 + 5x)(5x + 6y^2)^2$

8.3 Maxima and Minima

57. 10ft by 10ft by 10ft.

58. a. Max revenue when $x = 30$ logic games and $y = 15$ deluxe logic games.

b. Logic games @ \$15, and deluxe logic games @ \$165.