Week in Review # 7

MATH 142 — Drost-Spring 2010

Section 6.1, 6.2, 6.4 — April 4, 2010

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1. $\int (-4)\, dx$

$$\boxed{-4x + C}$$

2. $\int (2x)\, dx$

$$\boxed{x^2 + C}$$

3. $\int e^3\, dt$

$$\boxed{e^3 t + C}$$

4. $\int 24\sqrt{x}\, dx = \int 24x^{1/2}\, dx$

$$= 24 \cdot \frac{2}{3} x^{3/2} + C$$

$$= \boxed{16x^{3/2} + C}$$

5. $\int (x - e^x)\, dx$

$$\boxed{\frac{1}{2}x^2 - e^x + C}$$

6. $\int x^4 (4 + x^3)\, dx =$

$$\int (4x^4 + x^7)\, dx =$$

$$\boxed{\frac{4}{5}x^5 + \frac{1}{8}x^8 + C}$$

7. $\int \frac{5}{t^2}\, dt =$

$$\int 5t^{-2}\, dt$$

$$\frac{5t^{-1}}{-1} + C$$

$$\boxed{-\frac{5}{t} + C}$$

8. $\int \frac{1 - x^3}{x^4}\, dx =$

$$\int \left(\frac{1}{x^4} - \frac{x^3}{x^4}\right) dx$$

$$\int \left(x^{-4} + \frac{1}{x}\right) dx$$

$$\boxed{\frac{1}{-3}x^{-3} + \ln|x| + C}$$

9. $\int \left(\frac{3}{x^4} - \frac{12}{x^2} + 1 \right) dx$

$\int (3x^{-4} - 12x^{-2} + 1) dx$

$\frac{3x^{-3}}{-3} - \frac{12x^{-1}}{-1} + x + C$

$\boxed{\frac{-1}{x^3} + \frac{12}{x} + x + C}$

10. $\int \frac{1 - xe^x}{x} dx =$

$\int \left(\frac{1}{x} - e^x \right) dx$

$\boxed{\ln|x| - e^x + C}$

11. *Sales analysis.* Monthly sales of a particular personal computer are expected to decline at the rate of

$$S'(t) = -25t^{\frac{2}{3}}$$

computers per month, where t is time in months and $S(t)$ is the number of computers sold each month. The company plans to stop manufacturing this computer when monthly sales reach 800 computers. If monthly sales now $(t = 0)$ are 2000 computers, find $S(t)$. How long will the company continue to manufacture this computer? *Source: CALCULUS, by Barnett, p.371, #111*

$S' = -25t^{2/3}$

$S = \int -25t^{2/3} dt$

$S = -25\left(\frac{3}{5}\right) t^{5/3} + C$

$S = -15t^{5/3} + C$

$S(0) = 2000$

$2000 = -15t^0 + C$

$2000 = -15 + C$

$2015 = C$

$S = -15t^{5/3} + 2015$

Stop when $S = 800$

$800 = -15t^{5/3} + 2015$

$15t^{5/3} = 1215$

$t^{5/3} = 81$

$t = 13.966\ldots$

$\boxed{\text{at 14 months}}$

12. The rate of change of salary for a minor leage ballplayer is modeled by $-400 + 1500\sqrt{x}$ over the interval $[1, 16]$ with $S(1) = 1100$. Find the ballplayer's salary his 10^{th} year.

$$S' = -400 + 1500\sqrt{x}$$

$$S' = -400 + 1500x^{1/2}$$

$$S = \int(-400 + 1500x^{1/2})dx$$

$$S = -400x + 1500 \cdot \frac{2}{3}x^{3/2} + C$$

$$S = -400x + 1000x^{3/2} + C$$

$$S(1) = -400(1) + 1000(1) + C = 1100$$

$$600 + C = 1100$$

$$C = 500$$

$$S(x) = -400x + 1000x^{3/2} + 500$$

$$S(10) = -400(10) + 1000(10)^{3/2} + 500$$

$$S(10) \approx 28{,}122.7766\ldots$$

$$\boxed{\$28{,}122.78}$$

13. A bacteria culture is growing at the rate $W'(t) = 0.4e^{0.1t}$ grams per hour. If the culture weighed 2 grams originally, what is the weight of the culture $W(t)$

a. after t hours?

b. after 90 minutes?

$$W' = 0.4e^{0.1t}$$

$$W = \int 0.4e^{0.1t}\,dt$$

$$W = \frac{0.4}{.1}\int e^{0.1t}(.1)\,dt$$

$$W = 4e^{0.1t} + C$$

a)

$$W(0) = 4e^{0} + C = 2$$

$$4 + C = 2$$

$$C = -2$$

$$\boxed{W(t) = 4e^{0.1t} - 2}$$

b) $W(1.5) = 4e^{0.1(1.5)} - 2 = \boxed{2.6473 \text{ grams}}$

14. $\int \frac{1}{2x+5}\,dx = \int \frac{1}{u}\cdot\frac{1}{2}\,du = \frac{1}{2}\int \frac{1}{u}\,du$

let $u = 2x+5$

$$\frac{du}{dx} = 2$$

$$du = 2dx$$

$$\frac{1}{2}du = dx$$

$$= \frac{1}{2}\ln|u| + C$$

$$= \boxed{\frac{1}{2}\ln|2x+5| + C}$$

re-worked with Mrs Drost's fix-it method

$$\int \frac{\text{mess}'}{\text{mess}}\,dx = \ln|\text{mess}| + C$$

$$\int \frac{1}{2x+5}\,dx$$

is the numerator mess' or do we need to "fix it"

$$\frac{1}{2}\cdot\int \frac{1\cdot 2}{2x+5}\,dx$$

$$\boxed{\frac{1}{2}\ln|2x+5| + C}$$

15. $\int e^{-x^2} \cdot x\, dx$

$\int e^{mess} \cdot mess'\, dx = e^{mess} + C$

$-\frac{1}{2}\int e^{-x^2}(-2x)\,dx$ fix-it

$$\boxed{-\frac{1}{2}e^{-x^2} + C}$$

One more time with Substitution method

$\int e^{-x^2} \cdot x\,dx =$

substituting

$\int e^{u} \cdot \left(-\frac{1}{2}\right) du =$

$-\frac{1}{2}\int e^{u}\,du =$

$-\frac{1}{2}e^{u} + C = \boxed{-\frac{1}{2}e^{-x^2} + C}$

$u = -x^2$

$\frac{du}{dx} = -2x$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

16. $\int 12x^2\sqrt{x^3+1}\,dx =$

$\int (x^3+1)^{1/2}(12x^2\,dx)$

$\frac{12}{3}\int \underbrace{(x^3+1)}_{mess}{}^{1/2}\underbrace{(3x^2)}_{mess'}dx$

$mess' = 3x^2$

$4 \cdot \frac{2}{3}(x^3+1)^{3/2} + C$

$$\frac{8}{3}(x^3+1)^{3/2} + C$$

with substitution

$u = x^3+1$

$\frac{du}{dx} = 3x^2$

$du = 3x^2\,dx$

$4\,du = 12x^2\,dx$

$\int (u)^{1/2} \cdot 4\,du$

$4 \cdot \frac{2}{3}u^{3/2} + C$

$$\frac{8}{3}(x^3+1)^{3/2} + C$$

17. $\displaystyle\int \frac{x}{\sqrt{x+12}}\,dx = \int \underbrace{(x+12)}_{\text{meso}}^{-1/2}(x)\,dx$

meso$' = 1 \neq x$

therefore do not use "fix-it"

$u = x+12$

$u-12 = x$

$du = dx$

$= \displaystyle\int u^{-1/2}(u-12)\,du$

$= \displaystyle\int u^{1/2} - 12u^{-1/2}\,du$

$= \frac{2}{3}u^{3/2} - 12\cdot 2u^{1/2} + C$

$$\boxed{= \frac{2}{3}(x+12)^{3/2} - 24(x+12)^{1/2} + C}$$

18. $\displaystyle\int x\,\underbrace{(x+2)}^{3}\,dx = \int (x+2)^3 \cdot x\,dx =$

$u = x+2$

$\frac{du}{dx} = 1$

$du = dx$

$u-2 = x$

$\displaystyle\int u^3(u-2)\,du =$

$\displaystyle\int (u^4 - 2u^3)\,du =$

$\frac{1}{5}u^5 - \frac{2}{4}u^4 + C$

$$\boxed{\frac{1}{5}(x+2)^5 - \frac{1}{2}(x+2)^4 + C}$$

19. $\displaystyle\int \frac{x^2+2x}{(x^3+3x^2+2)^4}\,dx =$ "fix-it"

$$\frac{1}{3}\int \underbrace{(x^3+3x^2+2)}_{\text{meso}}{}^{-4}\,3(x^2+2x)\,dx$$

$\text{meso}' = 3x^2+6x$

$$= \frac{1}{3}\cdot\frac{1}{-3}(x^3+3x^2+2)^{-3}+C = \boxed{-\frac{1}{9}(x^3+3x^2+2)^{-3}+C}$$

$u = x^3+3x^2+2$

$\frac{du}{dx} = 3x^2+6x$

$\frac{1}{3}du = \frac{(3x^2+6x)dx}{3}$

$\frac{1}{3}du = (x^2+2x)dx$

$\displaystyle\int \frac{x^2+2x}{(x^3+3x^2+2)^4}\,dx =$

$\displaystyle\frac{1}{3}\int u^{-4}\cdot du =$

$\frac{1}{3}\cdot\frac{1}{-3}u^{-3}+C$

$\boxed{-\frac{1}{9}(x^3+3x^2+2)^{-3}+C}$

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20. $\displaystyle\int \frac{(\ln x)^3}{x}\,dx = \int \underbrace{(\ln x)^3}_{\text{meso}}\cdot\underbrace{\frac{1}{x}}_{\text{meso}'}dx = \frac{1}{4}(\ln|x|)^4+C$

$\displaystyle\int \frac{(\ln x)^3}{x}\,dx = \int (\ln x)^3\cdot\frac{1}{x}\,dx$

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$du = \frac{1}{x}dx$

$\displaystyle= \int u^3\cdot du$

$= \frac{1}{4}u^4+C$

$= \boxed{\frac{1}{4}(\ln|x|)^4+C}$

21. $\int \frac{\sqrt{\ln x}}{x}\,dx = \int u^{1/2}\,du = \frac{2}{3}u^{3/2} + C$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$= \boxed{\frac{2}{3}(\ln|x|) + C}$

22. $\int \frac{x}{\sqrt[5]{2x^2+5}}\,dx = \frac{1}{4}\int \underbrace{(2x^2+5)}_{\text{mess}}^{-1/5} \cdot 4x\,dx$

"fix-it" mess$' = 4x$

$= \frac{1}{4}\cdot\frac{5}{4}(2x^2+5)^{4/5} + C$

$= \boxed{\frac{5}{16}(2x^2+5)^{4/5} + C}$

Substitution

$u = 2x^2+5$

$\frac{du}{dx} = 4x$

$du = 4x\,dx$

$\frac{1}{4}du = x\,dx$

$\int (2x^2+5)^{-1/5} \cdot x\,dx =$

$\frac{1}{4}\int u^{-1/5} \cdot du =$

$\frac{1}{4}\cdot\frac{5}{4}u^{4/5} + C$

$\boxed{\frac{5}{16}(2x^2+5)^{4/5} + C}$

23. $\int 5x\,e^{4-x^2}\,dx = \frac{5}{-2}\int e^{\overbrace{4-x^2}^{\text{mess}}} \cdot (-2)x\,dx$

mess$' = -2x$

$= \boxed{-\frac{5}{2}e^{4-x^2} + C}$

note: $\int e^{\text{mess}} \cdot \text{mess}'\,dx = e^{\text{mess}} + C$

$\int e^{4-x^2} \cdot 5x\,dx = \int e^u\left(-\frac{5}{2}\right)du$

$u = 4-x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$-\frac{5}{2}du = 5x\,dx$

$= -\frac{5}{2}e^u + C$

$= \boxed{-\frac{5}{2}e^{4-x^2} + C}$

24. Find the revenue function for a purse manufacturer if the marginal revenue is given by $(1-x)e^{2x-x^2}$, where x is the number of thousands of purses sold. ans: $\frac{1}{2}e^{2x-x^2}-\frac{1}{2}$

$MR = R' = e^{2x-x^2}(1-x)$

$R = \int e^{2x-x^2}(1-x)\,dx = \int e^u \cdot \frac{1}{2}\,du =$

using sub. $u = 2x - x^2$

$\frac{1}{2}du = \frac{(2-2x)}{2}dx$

$\frac{1}{2}du = (1-x)\,dx$

$R = \frac{1}{2}e^u + C$

$R = \frac{1}{2}e^{2x-x^2} + C$

$R(0) = 0$

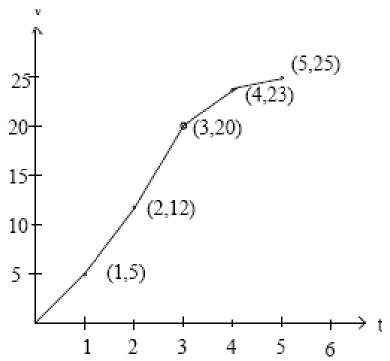
$0 = \frac{1}{2}e^0 + C$

$0 = \frac{1}{2} + C$

$-\frac{1}{2} = C$

$$\boxed{R = \frac{1}{2}e^{2x-x^2} - \frac{1}{2}}$$

25. An object travels with a velocity function given in the following figure. Find an upper and lower estimate of the distance traveled by the object.



$t=$	0	1	2	3	4	5
$d=$	0	5	12	20	23	25

$\Delta x = \frac{5-0}{5} = 1$

$L_5 = 1(0+5+12+20+23)$

$\boxed{L_5 = 60 \text{ units}}$

$R_5 = 1(25+23+20+12+5)$

$\boxed{R_5 = 85 \text{ units}}$

26. If $f(x) = -(x-4)^2 + 20$, approximate the area under the curve $f(x)$ and above the x-axis from $x = 0$ to $x = 6$, using 3 rectangles from

a. the left.

b. the right.

x	0	1	2	3	4	5	6
$f(x)$	4	11	16	19	20	19	16

$\Delta x = \dfrac{6-0}{3} = 2$

$L_3 = 2(f_0 + f_2 + f_4) =$

$L_3 = 2(4 + 16 + 20) = \boxed{80 \text{ units}^2}$

$R_3 = 2(f_6 + f_4 + f_2)$

$R_3 = 2(16 + 20 + 16) = \boxed{104 \text{ units}^2}$

27. How many rectangles should be used in the previous problem so that the error is less than 1?

$\text{Error} \le |f(b) - f(a)| \cdot \dfrac{b-a}{n}$

$\text{Error} \le 1$

$|f(b) - f(a)| \cdot \dfrac{b-a}{n} = 1$

$|f(6) - f(0)| \cdot \dfrac{6-0}{n} = 1$

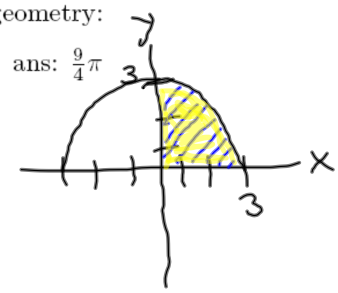
$|16 - 4| \cdot 6 = n$

$12 \cdot 6 = n$

$\boxed{n = 72}$

28. Find the exact value using geometry: $\displaystyle\int_0^3 \sqrt{9 - x^2}\, dx$

ans: $\frac{9}{4}\pi$



$f(x) = \sqrt{9 - x^2}$

$y = \sqrt{9 - x^2}$

$y^2 = 9 - x^2$

$x^2 + y^2 = 9$

circle, center $(0,0)$

radius $= 3$

$y = +\sqrt{}$ is upperhalf

$A = \frac{1}{4}\pi r^2$

$A = \frac{1}{4}\pi (3)^2$

$\boxed{A = \frac{9}{4}\pi \text{ units}^2}$

29. From a study on memorizing facts, a researcher found on average, the rate of learning facts, $N'(x)$ after x hours of studying was defined approximately by the following values:

x	0	2	4	6	8	10	12
$N'(x)$	30	26	22	20	18	16	12

$\Delta x = \frac{12-6}{3} = 2$

Use left and right sums over three equal subintervals to approximate the area under the graph of $N'(x)$ from $x = 6$ to $x = 12$. Calculate the error bound.

$L_3 = 2(20+18+16)$

$L_3 = 2(54)$

$L_3 = 108$

Fill in the blanks:

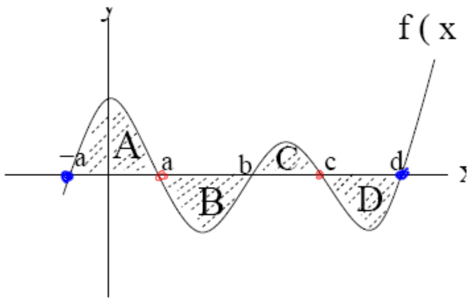
$$\boxed{92 \leq \int_6^{12} N'(x)\,dx \leq 108}$$

$R_3 = 2(12+16+18)$

$R_3 = 2(46) = 92$

30. Evaluate the definite integrals below given:

$A = 2.4, B = 1.25, C = 0.5, D = 1.5$



a) $\int_{-a}^{d} f(x)\,dx =$

$A - B + C - D =$

$2.4 - 1.25 + 0.5 - 1.5 =$

$2.90 - 2.75 = \boxed{0.15}$

b) $\int_a^c f(x)\,dx =$

$-B + C =$

$-1.25 + 0.5 =$

$\boxed{-.75}$

c) $\int_b^c f(x)\,dx = C$

$= \boxed{0.5}$

d) $\int_d^b f(x)\,dx =$

$-\int_b^d f(x)\,dx =$

$-[C - D] =$

$-C + D =$

$-.5 + 1.5 = \boxed{1}$

Use the Properties of definite integrals, and

$$\int_0^3 x\,dx = 4.5, \int_0^3 x^2\,dx = 9, \int_3^6 x^2\,dx = 63$$

to evaluate each of the following:

31. $\int_0^3 6x\,dx = 6\int_0^3 x\,dx$

$= 6(4.5)$

$= \boxed{27}$

32. $\int_0^3 4x^2\,dx =$

$4\int_0^3 x^2\,dx =$

$4 \cdot 9 = \boxed{36}$

33. $\int_3^3 6x^2\,dx = \boxed{0}$

34. $\int_6^3 5x^2\,dx = -5\int_3^6 x^2\,dx$

$= -5(63)$

$= \boxed{-315}$

35. $\int_0^3 (5x + 3x^2)\,dx =$

$5\int_0^3 x\,dx + 3\int_0^3 x^2\,dx =$

$5(4.5) + 3(9) =$

$22.5 + 27 =$

$\boxed{49.5}$