

Math 365 Lecture Notes Section 2.2 – Other Set Operations and Their Properties

★ Set Operations

Definitions:

- 1) Intersection of two sets is the set of all elements contained in both sets. $A \cap B$

What is the intersection between the set of rhombuses and the set of rectangles?

- 2) Union – The union of two sets A and B ($A \cup B$) is the set of all elements that are either members of A, or B, or both.

What is the union between the set of people with a learner’s permit to drive and the set of people with a driver’s license?

- 3) Complement of A relative to B, written B-A, is the set of all elements in B that are not in A.

Problem 1: The sets A, B, and C are defined below. Use these sets to find each of the following and, if possible, shade a Venn diagram to represent each problem.

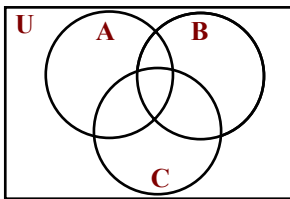
$$A = \{p, r, o, f, e, s\}$$

$$B = \{d, o, n, t, l, i, k, e\}$$

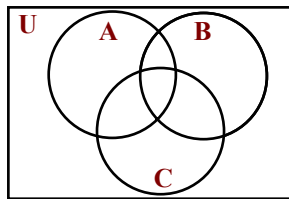
$$C = \{c, h, e, a, t, r, s\}$$

$$U = \{x \mid x \text{ is a letter of the alphabet that precedes or includes the letter } t\}$$

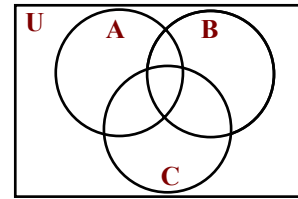
a) $A \cap B$



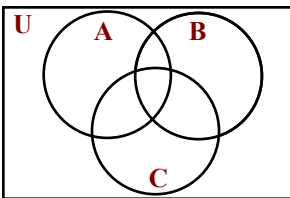
b) $(A \cup B) \cap C$



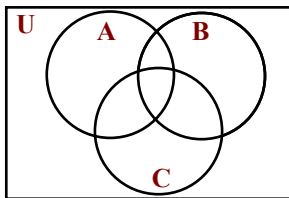
c) $n(A \cup C)$



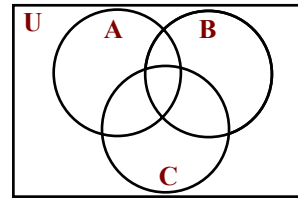
d) $A^c \cap C^c$



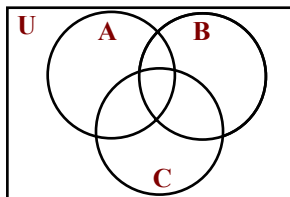
e) $A - B$



f) $(C - B) \cap A$



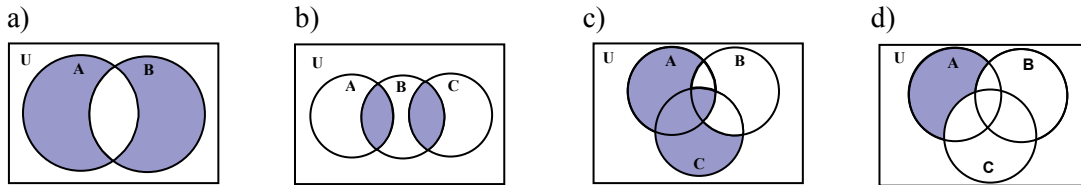
g) $B^c \cup (A - C)$



★ **Properties of Set Operations**

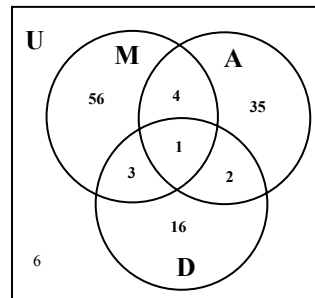
- 1) Commutative Property of Set Union:
- 2) Commutative Property of Set Intersection:
- 3) Distributive Property of Set Intersection over Union:

Problem 2: Use set notation to describe the shaded portions of the Venn diagrams below.

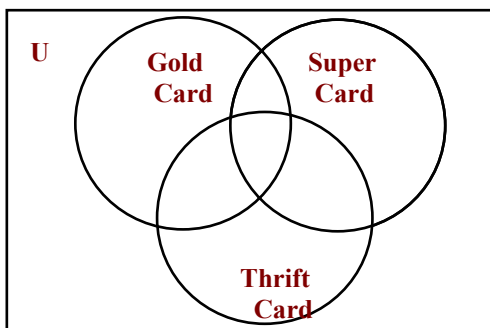


Problem 3: In the Venn diagram below, let M = students who take a music class, A = students who take an art class, and D = students who take a drama class. Find each of the following and then describe (in words) what each of the following represent.

- a) $n(M - A)$
- b) $n(A)$
- c) $n(M^c \cap D)$
- d) $n(M \cup A \cap D^c)$
- e) $n(M \cap A \cap D)$
- f) $n(M \cap A \cap D^c)$
- g) $n(M \cup D \cup A)$



Problem 4: A pollster interviewed 500 university seniors who owned credit cards. She reported that 240 owned Goldcard, 290 had Supercard, and 270 had Thriftcard. Of those seniors, the report said that 80 owned only a Goldcard and a Supercard, 70 owned only a Goldcard and a Thriftcard, 60 owned only a Supercard and a Thriftcard, and 50 owned all 3 cards. When the report was submitted for publication in the local campus newspaper, the editor refused to publish it, claiming the poll was not accurate. Was the editor right? Why or why not?



Definitions:

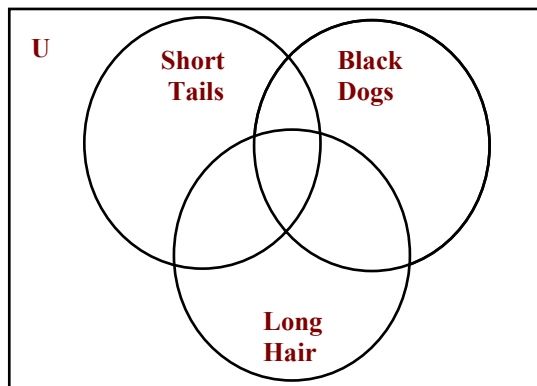
- 1) Cartesian Product:

Problem 5: If $A = \{a, b\}$ and $B = \{1, 2, 3\}$, find $B \times A$, and $A \times A$.

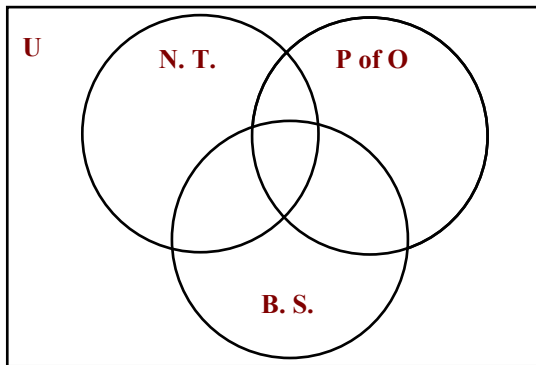
Problem 6: Given $n(A) + n(B) = n(A \cup B)$, then find $n(A \cap B)$.

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Problem 7: Twenty four dogs are in a kennel. Twelve of the dogs are black, six of the dogs have short tails, and fifteen of the dogs have long hair. There is only one dog that is black with a short tail and long hair. Two of the dogs are black with short tails and do not have long hair. Two of the dogs have short tails and long hair but are not black. If all of the dogs in the kennel have at least one of the characteristics, how many dogs are black with long hair but do not have short tails?
Source: <http://regentsprep.org/Regents/Math/venn/LVenn.htm>



Problem 8: In a survey of 100 students,
70 had seen National Treasure;
15 had seen Phantom of the Opera;
48 had seen Bourne Supremacy.
28 had seen National Treasure and Bourne Supremacy.
7 had seen Phantom of the Opera and National Treasure.
5 had seen all three.
30 had seen at least two of the movies.
How many of the students had seen exactly two of the movies?
How many of the students had seen none of these movies?



Problem 9: The students at Pleasant Hill Elementary School (in Oklahoma City, OK) can choose 1 morning class from $M = \{\text{German, Spanish, French}\}$ and one afternoon class from $A = \{\text{Robotics, Logic, Physics, Photography}\}$. Find $M \times A$.