Math 365 Lecture Notes
Section 3.1 – Numeration Systems

☆Numeration Systems

What are some cultural properties that would influence the development of numeration systems?

Definitions:

1) Numeration System: a collection of properties and symbols to represent numbers

2) Numerals: the symbols 3, and 8 are numerals

Types of Numeration Systems:

1) Additive: the value of a number was the sum of the face values of the numerals

2) Place Value: the value of a number depends on its placement in a numeral

☆Tally Numeration System (additive)

	one	two	three	four	five	six	seven	eight	nine	ten
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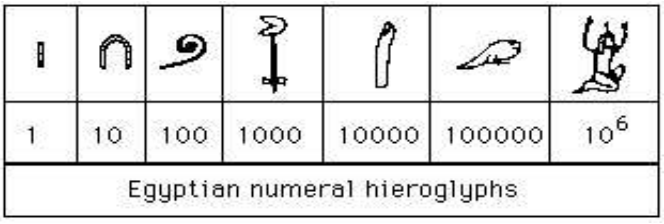
Discussion Questions

1) What are some advantages to this numeration system?

2) What are some disadvantages to this numeration system?

☆Egyptian Numeration System (additive)



1	10	100	1000	10000	100000	10^6

Egyptian numeral hieroglyphs

Problem 1: Use the Egyptian system to represent 1254.

☆Babylonian Numeration System (place value – base 60)

What numerals are used in the Babylonian system and what are the face values of the numerals?

Problem 2: Find the value of each Babylonian number.

1) ▼▼▼ <<

$\_\_\_\_ \cdot 60^1 + \_\_\_\_ \cdot 60^0$

2) <▼ <<<▼ <▼▼▼▼

$\_\_\_\_ \cdot 60^2 + \_\_\_\_ \cdot 60^1 + \_\_\_\_ \cdot 60^0$

3) ▼▼▼ ▼▼ <<▼

$\_\_\_\_ \cdot 60^2 + \_\_\_\_ \cdot 60^1 + \_\_\_\_ \cdot 60^0$

☆Mayan Numeration System (place value – modified base 20)

What numerals are used in the Mayan system and what are the face values of the numerals?

In a true base 20 system, the third position vertically from the bottom should be $20^2 = 400$, but this is NOT the case in the Mayan number system. Instead the Mayan's used

20^0, 20^1, $20^1 \cdot 18$, $20^2 \cdot 18$, $20^3 \cdot 18$, $20^4 \cdot 18$, …

Example of Mayan number:

Problem 3: Convert the following Mayan numbers to Hindu-Arabic (in expanded form).

a)

b)

For an explanation of why they chose this number system go to the following website
http://www-groups.dcs.st-andrews.ac.uk/~history/HistTopics/Mayan_mathematics.html

☆Roman Numeration System (additive/subtractive/multiplicative)

What numerals are used in the Roman system and what are the face values of the numerals?

Important properties of Roman numeral system:

1) Additive Property: A property in which the face value of the numerals are added together using the largest number possible at each stage (i.e. 15 is written XV not VVV). No numeral is written more than 3 consecutive times.

2) Subtraction Property

a) Only I, X, and C can be used to subtract.

b) Only one smaller number can be placed to the left.

c) The subtracted value must be no less than a tenth of the value of the number it is subtracted from. So the only numbers obtained through the subtraction property are:

3) Multiplicative Property

$\overline{CDX} =$

$\overline{\overline{CXI}} =$

$\overline{CX}\,I =$

☆Hindu-Arabic Numeration System (place value – base 10)

What numerals are used in the Hindu-Arabic system?

What are the first 5 place values of the Hindu-Arabic system?

Write 5,984 in expanded form.

☆Other Base Systems – Introductory Activity

1. Get a "place-value mat" and thirty-seven pennies.
2. Take the thirty-seven pennies and make as many stacks of ten as possible out of these pennies. After each stack of ten pennies is made, place the stack on the "place-value mat" in the spot for $(\text{base})^1$.
3. Place any left over pennies on the "place-value mat" in the spot for $(\text{base})^0$.
4. Since there are 3 stacks of "tens" and 7 "ones", we write 37 to represent the pennies in base 10.
5. The pennies can also be grouped in 3 stacks of eleven and 4 single pennies. Thus we can write this number as 34_{eleven}.
6. By grouping your pennies as demonstrated in the previous two examples, write the number in base two through base twelve.

37 = _____two 37 = _____seven 37 = _____twelve

37 = _____three 37 = _____eight

37 = _____four 37 = _____nine

37 = _____five 37 = _____ten

37 = _____six 37 = _____eleven

☆Other Number Base Systems (place value)

Base	Digits Used	Example of Converting to Base 10
2		1011_{two} =
4		213_{four} =
6		250_{six} =
12		$TE39_{twelve}$ =

Problem 4: Write the first 10 natural numbers in base 2.

Problem 5: Change 824 to base five.

Problem 6: In base twelve, what five numbers come after 19?