

Math 365 Lecture Notes

Section 3.2 – Algorithms for Whole-Number Addition and Subtraction

★ Addition Algorithms

1. Concrete Model

2. Expanded Algorithm

$$\begin{array}{r} 23 \\ + 12 \\ \hline 5 \text{ (Add ones)} \\ + 30 \text{ (Add tens)} \\ \hline 35 \end{array}$$

3. Standard Algorithm

$$\begin{array}{r} 23 \\ + 12 \\ \hline 35 \end{array}$$

For combinations larger than nine, in elementary grades, use the words “regroup” or “trade” instead of “carry” to explain the process.

$$4. \quad \begin{array}{r} 46 \\ + 35 \\ \hline 11 \text{ (Add ones)} \\ + 70 \text{ (Add tens)} \\ \hline 81 \end{array} \qquad \begin{array}{r} 46 \\ + 35 \\ \hline 81 \end{array}$$

★ Adding in Other Bases

5. Get a “place-value mat” and some pennies.
6. On the mat, place the pennies to model $23_{\text{four}} + 33_{\text{four}}$.
7. To add these numbers we must regroup because there are too many single units for our base. So $23_{\text{four}} + 33_{\text{four}} =$

Problem 1: Use the “place value mat” and pennies to find the following sums.

- | | |
|--------------------------------------------|--------------------------------------------|
| a) $12_{\text{four}} + 22_{\text{four}} =$ | d) $42_{\text{five}} + 14_{\text{five}} =$ |
| b) $33_{\text{four}} + 22_{\text{four}} =$ | e) $13_{\text{five}} + 24_{\text{five}} =$ |
| c) $21_{\text{four}} + 33_{\text{four}} =$ | f) $41_{\text{five}} + 40_{\text{five}} =$ |

Problem 2: Use the standard algorithm to find the following sums.

- | | | |
|------------------------------------------------------------------------------------|------------------------------------------------------------------------------------|------------------------------------------------------------------------------------|
| a) $\begin{array}{r} 12_{\text{four}} \\ + 22_{\text{four}} \\ \hline \end{array}$ | b) $\begin{array}{r} 33_{\text{four}} \\ + 22_{\text{four}} \\ \hline \end{array}$ | c) $\begin{array}{r} 21_{\text{four}} \\ + 33_{\text{four}} \\ \hline \end{array}$ |
|------------------------------------------------------------------------------------|------------------------------------------------------------------------------------|------------------------------------------------------------------------------------|

$$\text{d) } \begin{array}{r} 42_{\text{five}} \\ + 14_{\text{five}} \\ \hline \end{array}$$

$$\text{e) } \begin{array}{r} 13_{\text{five}} \\ + 24_{\text{five}} \\ \hline \end{array}$$

$$\text{f) } \begin{array}{r} 41_{\text{five}} \\ + 40_{\text{five}} \\ \hline \end{array}$$

Problem 3: Use the expanded algorithm to find the following sums.

$$\text{a) } \begin{array}{r} 12_{\text{four}} \\ + 22_{\text{four}} \\ \hline \end{array}$$

$$\text{b) } \begin{array}{r} 33_{\text{four}} \\ + 22_{\text{four}} \\ \hline \end{array}$$

$$\text{c) } \begin{array}{r} 21_{\text{four}} \\ + 33_{\text{four}} \\ \hline \end{array}$$

$$\text{d) } \begin{array}{r} 42_{\text{five}} \\ + 14_{\text{five}} \\ \hline \end{array}$$

$$\text{e) } \begin{array}{r} 13_{\text{five}} \\ + 24_{\text{five}} \\ \hline \end{array}$$

$$\text{f) } \begin{array}{r} 41_{\text{five}} \\ + 40_{\text{five}} \\ \hline \end{array}$$

★ **Subtracting in Other Bases**

1. Get a “place-value mat” and some pennies. We will use the mat, to model $21_{\text{four}} - 13_{\text{four}}$. To do this lay out enough pennies to represent 21_{four} .
2. To subtract these numbers we must regroup because we can not take 3 units from 1 unit. So we take a stack of four pennies and give them to the ones place.
3. Now take away 1 stack of 4 and 3 single pennies.
4. So $21_{\text{four}} - 13_{\text{four}} =$

Problem 4: Use the “place value mat” and pennies to find the following differences.

$$\text{a) } 31_{\text{four}} - 12_{\text{four}} =$$

$$\text{c) } 42_{\text{five}} - 14_{\text{five}} =$$

$$\text{b) } 20_{\text{four}} - 3_{\text{four}} =$$

$$\text{d) } 31_{\text{five}} - 22_{\text{five}} =$$

Problem 5: Use the standard algorithm to find the following differences.

$$\text{a) } \begin{array}{r} 31_{\text{four}} \\ - 12_{\text{four}} \\ \hline \end{array}$$

$$\text{b) } \begin{array}{r} 20_{\text{four}} \\ - 3_{\text{four}} \\ \hline \end{array}$$

$$\text{c) } \begin{array}{r} 42_{\text{five}} \\ - 14_{\text{five}} \\ \hline \end{array}$$

$$\text{d) } \begin{array}{r} 31_{\text{five}} \\ - 22_{\text{five}} \\ \hline \end{array}$$

☆ **Other Algorithms for Adding Whole Numbers (base 10)**

1) Left-to-Right Algorithm for Addition:

$$\begin{array}{r} 587 \\ + 632 \\ \hline \end{array}$$

2) Lattice Algorithm for Addition

$$\begin{array}{r} 5 \ 4 \ 3 \ 7 \\ + 2 \ 8 \ 7 \ 5 \\ \hline \boxed{} \boxed{} \boxed{} \boxed{} \boxed{} \end{array}$$

3) Scratch Algorithm for Addition

$$\begin{array}{r} 94 \\ 57 \\ 31 \\ + 24 \\ \hline \end{array}$$

4) Equal Addends Algorithm