

## Math 365 Lecture Notes Section 3.3 – Algorithms for Whole-Number Multiplication and Division

### ★ Multiplying by Whole Numbers

1. Using the distributive property:

$$\begin{aligned}4 \times 12 &= \\4 \times (10 + 2) &= \\4 \times 10 + 4 \times 2 &= 40 + 8 = 48\end{aligned}$$

2. Multiplying with Zeros

$$\begin{aligned}5 \cdot 3,000 &= 5 \cdot (3 \cdot 10^3) \\&= (5 \cdot 3) \cdot 10^3 \\&= 15 \cdot 10^3 \\&= 15,000\end{aligned}$$

$$\begin{aligned}5000 \cdot 700 &= \\5 \cdot 10^3 \cdot 7 \cdot 10^2 &= \\5 \cdot 7 \cdot 10^3 \cdot 10^2 &= \\35 \cdot 10^5 &= 3,500,000\end{aligned}$$

3. Is the Answer Reasonable?

$$14 \rightarrow 23 = ???$$

$$10 \cdot 20 = 200$$

$$14 \rightarrow 23 = ???$$

$$20 \cdot 30 = 600$$

4. Concrete Models

5. Lattice Multiplication



### ★ Multiplying by a Single Digit in Other Bases

6. Get a “place-value mat” and some pennies.
7. On the mat, place the pennies to model  $12_{\text{four}} \times 3_{\text{four}}$ .
8. To multiply these numbers we must regroup because there are too many single units for our base. So  $12_{\text{four}} \times 3_{\text{four}} =$

**Problem 1: Use the “place value mat” and pennies to find the following products.**

a)  $13_{\text{four}} \times 3_{\text{four}} =$

d)  $42_{\text{five}} \times 2_{\text{five}} =$

b)  $33_{\text{four}} \times 2_{\text{four}} =$   
 c)  $21_{\text{four}} \times 3_{\text{four}} =$

e)  $13_{\text{five}} \times 3_{\text{five}} =$   
 f)  $41_{\text{five}} \times 4_{\text{five}} =$

**Problem 2: Use the standard algorithm to find the following products.**

a) 
$$\begin{array}{r} 13_{\text{four}} \\ \times 3_{\text{four}} \\ \hline \end{array}$$

b) 
$$\begin{array}{r} 33_{\text{four}} \\ \times 2_{\text{four}} \\ \hline \end{array}$$

c) 
$$\begin{array}{r} 21_{\text{four}} \\ \times 3_{\text{four}} \\ \hline \end{array}$$

**Problem 3: Use the expanded algorithm to find the following products.**

a) 
$$\begin{array}{r} 42_{\text{five}} \\ \times 2_{\text{five}} \\ \hline \end{array}$$

b) 
$$\begin{array}{r} 13_{\text{five}} \\ \times 3_{\text{five}} \\ \hline \end{array}$$

c) 
$$\begin{array}{r} 41_{\text{five}} \\ \times 4_{\text{five}} \\ \hline \end{array}$$

★ **Multiplying by Two Digits in Other Bases**

**Problem 4: Use the standard algorithm to find the following products.**

a) 
$$\begin{array}{r} 31_{\text{four}} \\ \times 12_{\text{four}} \\ \hline \end{array}$$

b) 
$$\begin{array}{r} 42_{\text{five}} \\ \times 14_{\text{five}} \\ \hline \end{array}$$

c) 
$$\begin{array}{r} 31_{\text{five}} \\ \times 44_{\text{five}} \\ \hline \end{array}$$

★ **Building a Multiplication Table**

X	0	1	2	3	4
0					
1					
2					
3					
4					

Multiply  $231_{\text{five}} \cdot 122_{\text{five}}$

★ Short Division

★ Dividing in by a Single Digit in Other Bases

**Problem 5:** Use the standard division algorithm to find the following quotients.

a)  $4_{\text{five}} \overline{)1442_{\text{five}}}$

b)  $3_{\text{five}} \overline{)20312_{\text{five}}}$