

Math 365 Lecture Notes Section 4.3 – Divisibility

★ Notation, Definitions, and Theorems

- 1) Definition of “ b divides a ” –
- 2) Different ways to describe divisibility
 - a is divisible by b
 - b is a divisor of a
 - a is a multiple of b
 - b is a factor of a
 - b divides a

Problem 1: Fill in each blank with one of the following words/phrase.

Word Bank
factor/divisor
multiple
divides

- a) 5 is a _____ of 100.
 - b) 60 is a _____ of 3.
 - c) If $x \in W$, then $7x$ is a _____ of x .
 - d) Since 4 is a _____ of 20, 4 _____ 20.
- 3) **Theorem 4-2 (pg. 216 Billstein):** If $x, y, z \in I$ then the following are true.
- a) If $z \mid x$ and $z \mid y$, then $z \mid (x + y)$.
 - b) If $z \mid x$ and $z \mid y$, then $z \mid (x - y)$.
- 4) **Example Problems:**
- | | |
|-------------------------------|----------------------|
| a) $3 \mid 12$ | True or False |
| b) $0 \mid 5$ | True or False |
| c) $1 \mid a, a \in I$ | True or False |
| d) $4 \mid 12a, a \in I$ | True or False |
| e) $(a - b) \mid (a^2 - b^2)$ | True or False |
- $a, b \in I, a \neq b$

★ Divisibility Rules

- **Divisibility Test for 2:** An integer is divisible by 2 if, and only if, its units digit is divisible by 2.

Which of the following numbers are divisible by 2: 315, 268, 1319, 2580, or 1,236
- **Divisibility Test for 3:** An integer is divisible by 3 if, and only if, the sum of its digits is divisible by 3.

Which of the following numbers are divisible by 3: 315, 268, 1319, 2580, or 1,236

- **Divisibility Test for 4:** An integer is divisible by 4 if, and only if, the last two digits of the integer represent a number divisible by 4.

Which of the following numbers are divisible by 4: 315, 268, 1319, 2580, or 1,236

- **Divisibility Test for 5:** An integer is divisible by 5 if, and only if, its units digit is divisible by 5.

Which of the following numbers are divisible by 5: 315, 268, 1319, 2580, or 1,236

- **Divisibility Test for 6:** An integer is divisible by 6 if, and only if, the integer is divisible by both 2 and 3.

Which of the following numbers are divisible by 6: 315, 268, 1319, 2580, or 1,236

- **Divisibility Test for 8:** An integer is divisible by 8 if, and only if, the last three digits of the integer represents a number divisible by 8.

Which of the following numbers are divisible by 8: 312, 264, 1319, 2580, or 1,232

- **Divisibility Test for 9:** An integer is divisible by 9, if, and only if, the sum of the digits of the integer is divisible by 9.

Which of the following numbers are divisible by 9: 315, 268, 1319, 2580, or 1,236

- **Divisibility Test for 10:** An integer is divisible by 10, if, and only if, its units digit is divisible by 10.

Which of the following numbers are divisible by 10: 315, 268, 1319, 2580, or 1,236

- **Divisibility Test for 11:** An integer is divisible by 11 if, and only if, the sum of the digits in the places that are even powers of 10 minus the sum of the digits of the places that are odd powers of 10 is divisible by 11.

Which of the following numbers are divisible by 11: 31526, 26873, 1309, 2580, or 12366

Problem 2: Without a calculator, test the number 4,201,012 (four million, two hundred one thousand, twelve) for divisibility by 2, 3, 4, 5, 6, 8, 9, 10, and 11.

	Circle One	Test/Explanation
Two	Yes / no	
Three	Yes / no	
Four	Yes / no	
Five	Yes / no	
Six	Yes / no	
Eight	Yes / no	
Nine	Yes / no	
Ten	Yes / no	
Eleven	Yes / no	