Math 365 Lecture Notes
Section 6.3 : Non-terminating Decimals

☆Repeating Decimals

Decimal representations of numbers in general may terminate (like 0.5), may repeat forever (like $1/3 = 0.\overline{3} = 0.33333...$), or may do neither (i.e. go on forever without repeating, like the digits of $\pi = 3.1415926535...$).

Definition: the repeating block of digits is called the **repetend,** and is written with a bar over it.
Example: $.\overline{25} = .2525...$

On your own: Convert $\frac{1}{7}$ and $\frac{2}{13}$ to decimals using long division. Then, examine the remainders in each stage of the division process. Make conjectures as to why the digits repeat and how many digits may repeat.

Conjectures:
1. The digits repeat because ____

____

2. The number of digits in the repetend is ________________________________

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Examples: Convert the following decimals to the form $\frac{a}{b}$ where a, b are integers with $b \neq 0$.

a) $0.\overline{55}$ b) $0.\overline{235}$ c) $2.13\overline{29}$

Describe the process used to convert repeating decimals to a ratio of two nonzero integers.

Given your knowledge about the correspondence between fractions and repeating decimals, can you write $0.\overline{9} = 0.99999999...$ as a ratio of two nonzero integers?

Ordering Repeating Decimals

On your own: Order the following decimals from least to greatest.

$2.467,\ 2.\overline{467},\ 2.\overline{46},\ -2.\overline{674},\ -2.\overline{67},\ -2.674$

Explain the process you used above to order the decimals.

Sample Problems

1) Find a rational number in decimal form between $14.3\overline{5}$ and $14.3\overline{51}$.

2) Find the number **exactly** halfway between $1.\overline{9}$ and 2.25.

3) Find the 30^{th} digit in the decimal expansion of $\frac{5}{7}$?

4) Mentally change $8.\overline{137}$ into a mixed number.

5) True or False: $B = \frac{22}{7}$

6) Find the decimal representation for $\frac{3}{7}$ and $\frac{4}{7}$. What do you observe?