

Week in Review #13

MATH 365
Review for Final

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1. A store marks up the price of an item 50%, and then puts it on sale at 25% off the regular price. What percentage profit does the store actually make on the sale of the item?
2. Find the sum of the first 21 terms of an arithmetic sequence in which the eighteenth term is $31 + 117\sqrt{5}$ and the thirty-third term is $331 + 222\sqrt{5}$.
3. Fully simplify: $\sqrt{\frac{4x^2z^8}{5y^4z^{14}}}$
4. If a number is rational, write " $\in Q$ " and if a number is not rational, write " $\notin Q$ ". Circle the numbers that are in fractional form.
 - a. $\frac{-23}{11}$
 - b. $\frac{5\pi}{2}$
 - c. $5.\overline{345}$
 - d. -9.426
 - e. 5^{-4}
 - f. 8
 - g. $3e$
 - h. 0
5. Explain or show why the quotient $60.35 \div 0.6$ can be found by moving the decimal point to the right one place in each number and then finding the quotient of $603.5 \div 6$.
6. $1728^{\frac{-2}{3}} =$
7. What is 11% of 38?
8. Given a right triangle with legs of length $2\sqrt{3}$ and $5\sqrt{15}$, find the length of the hypotenuse.
9. What is the inverse of $p \rightarrow q$?
10. Insert three fractions between $\frac{5}{7}$ and $\frac{20}{567}$ so that the five numbers together constitute a geometric sequence.
11. The total for a meal at a local restaurant is \$72.76. Tax on the meal is 8.25%, plus you will need to leave a 15% tip, based on the price of the meal before taxes. Estimate the total bill, which includes the price of the meal, tax and tip. Show all numbers and calculations used. Your calculations must be easy to do mentally.

12. True or False.

a. If $a \mid b$ then b is a factor of a .

b. $\sqrt{(-a)^2} = -a$

c. A prime number is any number with exactly two distinct positive divisors.

d. If s is a non-zero rational number, $(x + y)^s = x^s + y^s$.

e. $|x + y| = |x| + |y|$

f. $\sqrt{(a + b)^4} = a^2 + 2ab + b^2$

g. The irrational numbers are closed under the operation of multiplication.

h. $3 \mid 768$

i. $x^w y^w = (xy)^{w^2}$, assume all expressions are defined.

j. If p is true and q is false, the truth value for $\sim(\sim p \vee q)$ is true.

k. $a^{-n} = \frac{-1}{a^n}$, where $a \neq 0$.

l. The multiplicative identity of the nonzero integer a is 0.

m. $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \frac{a^{\frac{1}{n}}}{b^{\frac{1}{n}}} = \left(\frac{a}{b}\right)^{\frac{1}{n}}$, assume all expressions are defined.

n. If p is a prime number $\sqrt{p^2}$ is irrational.

o. The four-steps to Poyla's problem-solving process are understanding the problem, devising a plan, carrying out the plan, and looking back.

p. $\frac{a}{b}$ is never the reciprocal for $\frac{b}{a}$.

q. $\pi = \frac{22}{7}$

r. If $\frac{a}{b}$ is rational, then $\frac{a}{b} > \frac{a^2}{b^2}$.

s. When you subtract you always get smaller numbers.

t. If $(A \cup B) \sim A$, then $B \subseteq A$.

13. If ___ divides a number, then 36 divides the number. Circle all that apply.

a. 6

b. 4 and 9

c. 3 and 12

d. 2 and 18

e. 72

14. Order the rational numbers below from least to greatest.

0.19

$\frac{1}{4}$

0.2

$\frac{2}{9}$

$0.24\overline{57}$

15. State whether the following are repeating (R) or terminating (T). Justify your answers.

a. $\frac{5}{8}$

b. $\frac{5}{22}$

c. $\frac{6}{120}$

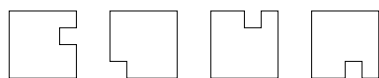
16. Explain how to compute $7.8 - 3.27$ and why your method works.

17. Fully simplify: $\sqrt[5]{\frac{-1024x^5y^8}{27x^{-15}z^{24}}} =$

18. Find the quotient $\frac{18.675}{7.5}$ by long division.

19. A store marks an item 30% off its regular price. A coupon is worth an extra discount of 10% off the sale price. What is the total percentage off the item's regular price?
20. Convert 36_{nine} to base four.
21. $\sqrt[n]{x^n} = \begin{cases} \underline{\hspace{2cm}} & \text{if } n \text{ is odd} \\ \underline{\hspace{2cm}} & \text{if } n \text{ is even} \end{cases}$
22. Compute 4.07×2.4 . Explain why you put the decimal point where you did in the product.
23. Find a rational number between $9.2\overline{56}$ and $9.2\overline{563}$.
24. Given $\frac{x^n}{y^m} = \left(\frac{x}{y}\right)^{n-m}$, where m , n , x , and y are integers and all expressions are defined. If this equality is true, write "true" and give an example illustrating this property. If this equality is false, write "false" and give a counterexample.
25. Find three irrational numbers between 6 and 7.
26. For which values of k can $\frac{k}{1400}$ be written as a terminating decimal? Explain your reasoning.
27. Eight-two and three hundredths is what percent of 157.75?
28. Write the following in simplest form (in the form of $a\sqrt[n]{b}$, where a and b are integers, and b has the least value possible).
- $\sqrt{1540}$
 - $\sqrt[3]{-1125}$
29. Write the following numerals in scientific notation.
- 0.006541
 - 2450000
30. Conjecture: If p is prime, \sqrt{p} is irrational.
- Proof: Suppose \sqrt{p} is rational. Then $\sqrt{p} = \frac{a}{b}$, where a is an integer and b is a non-zero integer by definition of _____. Thus $p = \frac{a^2}{b^2}$ by squaring both sides of the equation. Then $p \cdot b^2 = a^2$. Now a^2 has an _____ number of p 's in its prime factorization and pb^2 has an _____ number of p 's in its prime factorization. This is impossible since the _____ says that each composite number can be written as a product of primes in one, and only one, way except for the order of the prime factors. Therefore, by proof of contradiction, \sqrt{p} is irrational. qed
31. Solve: $5x\sqrt{2} - x\sqrt{2} = 7x\sqrt{2} + 5$
32. Model $3 + 2$
33. Prove or disprove: A number is divisible by 4 iff it is divisible by 8.
34. Respond to the conjecture, "Multiplication makes numbers greater."
- Under what circumstances is this conjecture true?
 - Under what circumstances is this conjecture false?

35. What is the additive inverse of $\frac{-2}{3} + \pi$?
36. Use two different models to compute $(-2) \cdot (-5)$.
37. A standard basketball court measures 50 feet by 94 feet. You wish to draw a scale diagram on a standard $8\frac{1}{2}$ by 11-inch sheet of notebook paper. If you want $\frac{1}{2}$ inch margins at the top and bottom of the paper, what should the dimensions of the scale drawing be? (There are two possible answers; I want the larger of the two.)
38. Use two different strategies of learning basic addition facts to add $6 + 7$.
39. Give an example of the associative property of addition of whole numbers.
40. What is the next figure in the sequence?



41. Write 849 in Roman numerals.
42. If $f(x) = 5x - 2$ and $g(x) = 9 - 3x$, find
- $f(4)$
 - $(f \circ g)(x)$
 - $g(f(2))$
43. Explain how you can remain in the whole numbers to make meaning of $24 \div 5$.
44. State whether the reasoning in each case is valid or invalid. If valid, indicate whether it is affirming the hypothesis or denying the conclusion. If invalid, indicate whether it is assuming the converse or assuming the inverse.
- If Jules solved this problem correctly, then Jules obtained the answer 2. Jules obtained the answer 2. Therefore, Jules solved this problem correctly.
 - If you live in College Station, then you live in Texas. You do not live in Texas. Therefore you do not live in College Station.
45. Model
- $\frac{1}{5} \div \frac{2}{3}$
 - $\frac{1}{5} \cdot \frac{2}{3}$
46. Give a model for showing the multiplication of $2 \cdot (3 + 4) = 2 \cdot 3 + 2 \cdot 4$ is valid.
47. $\sqrt[3]{\frac{27^n \cdot 3^{4n}}{9^{-n}}} =$
48. What is the multiplicative inverse of $\sqrt{6} - \frac{e}{2}$?
49. Use truth tables to show $[(p \rightarrow q) \wedge \sim q] \equiv \sim p \wedge \sim q$.
50. Use two different algorithms to compute $23 \cdot 34$.
51. Write the following in simplest form using positive exponents in the final answer. Assume all expressions are defined.

52. Kristen scored 45 goals during her soccer kicking practice. If her success-to-failure rate is 3:2, how many times did she attempt a goal?
53. Simplify, if possible. Write your answers in the form a^b or as a multiple thereof.
- $4^5 \cdot 4^2 \cdot 4^7 =$
 - $9^5 + 4 \cdot 9^5 =$
 - $8^{270} \cdot 8^{745} =$
54. $5 \oplus (4 \oplus 3) \bmod 8 =$
55. Given 3.45
- What is its expanded place value form?
 - What is its fractional meaning?
 - If a flat is one-tenth, model 3.45 with base-ten blocks.