

Week-in-Review, Week #12: Math 141, Spring 2004

1. 1a. Below is the probability distribution of the finite r.v., X . Fill in the missing number, and then answer the following questions.

x	2	4	10	15	20
$P(X = x)$	0.15	0.05	0.25	----	0.35

- 1b. Find the following measures of center: expected value (mean), median, mode.
1c. Find the “five number-summary”: minimum, Q_1 , median, Q_3 , maximum
1d. Find the following measures of spread: range, IQR (interquartile range), variance, standard-deviation
1e. Construct the cdf (“cumulative distribution function”) table.
1f. The random variable X is associated with a particular random experiment. If you perform three independent trials of this experiment, what is the probability that $X = 20$ in each trial?

2. A census of families was taken in a particular community, and the number of children in each family was noted.

The results: 38 families had no children, 32 families had one child, 22 families had two children, 18 families had three children, 6 families had 4 children, 1 family had 5 children, 2 families had 6 children, and no family had more than 6 children.

- 2a. Find the following summary measures for the “# of children” distribution: mean, median, mode, standard deviation, range, first quartile, third quartile.
2b. Suppose a family is selected at random. Let X be the number of children in that family. Set up the probability distribution table for X . Find the mean, median, mode, standard deviation, range, first quartile, and third quartile.
2c. If you pick four different families at random from the families in this community, what is the probability that exactly two of them have no children?
2d. If you pick one hundred families (with replacement!) from the families in this community and add up the number of children in those one hundred families, what is the expected total number of children?

3. Deal a five-card hand at random from the standard 52-card deck.

- 3a. Let X be the number of Kings in the hand. Give a formula for the probability distribution function (pdf). Present the probability distribution in tabular form.
3b. Find the expected value, median, mode, variance, standard deviation of X .
3c. A casino offers its customers the following game of chance: A five card hand is dealt, and the number of Kings in the hand is noted. The player receives \$1 for each King in the hand. If this is to be a fair game, how much should the casino charge for each play of the game?

- 4.** A jar contains ten nickels and five dimes. You reach into the jar and select three coins (without replacement) at random. Let the r.v., X = the total value of the three selected coins.
- Give the probability distribution of X (in tabular form).
 - Find the expected value, median, mode, range, variance, and std. dev. of X .
 - Suppose two hundred independent trials of this experiment will be conducted. Give your best guess for the total value of the coins drawn from the jar.
- 5.** The Acme Insurance Company offers a five-year term life insurance policy to people who have just turned 80. The insured buys the policy for a premium of \$15,000. If the insured dies within the five-year period, then the insurance company pays out \$50,000 to the beneficiary. (A fact from the insurance company's "life table": There is a 77.3% chance that a person who just turned eighty will still be alive five years later.)
- On the average, how much profit does the insurance company make on a policy?
 - The state's Insurance Regulatory Commission has just ruled that an insurance company's expected profit on a policy cannot exceed ten percent of the premium. Can the company keep charging a premium of \$15,000? If not, then what is the maximum which Acme can charge as the premium?
- 6.** Suppose that one word is to be selected at random from the ten words of the sentence, "To be or not to be, that is the question." Let X be the number of letters in the word selected.
- Set up the probability distribution of X in tabular form..
 - Sketch the histogram of X (Be sure to include labelled axes.)
 - Find the following summary measures:
 - mean
 - mode
 - median
 - standard deviation
 - variance
 - 75th percentile (or Q_3)
 - Why is the mean larger than the median?
- 7.** The weatherman on KDKA says the probability of rain today is 30%. According to the weatherwoman on KQED, the probability of rain is only 20%.
- According to the KDKA weatherman, what are the odds in favor of rain? What are the odds against rain?
 - According to the KQED weatherwoman, what are the odds in favor of rain? What are the odds against rain?
- 8.** The horse, Aggie's Delight is running in the fifth race at Rockingham Park. My bookie is posting odds of 7 to 5 against Aggie's Delight winning the race. According to my bookie, what is the probability that Aggie's Delight will win the race?

9. Later this season, California University (of Pa.) plays Indiana University (of Pa.) in football. Al and Bob make a bet as regards the outcome of this game. If California wins, Al pays Bob \$3. If Indiana wins, Bob pays Al \$5.

9a. Bob and Al agree that this is a fair bet. What is their common assessment of the probability that California University (of Pa.) wins?

9b. A common friend, Carl, tells Bob: "You're crazy to make this bet. It isn't a fair bet at all. The bet favors Al. According to my judgement/calculations, your expected losses are \$3." Bob asks, "How can I make the bet fair?" Carl answers, "If Indiana wins, you still pay Al five dollars. But if California wins, Al pays you \$_____."

According to Carl, how much should Al pay Bob if California wins?

10. Your company is considering the construction of a new assembly plant. You have 3 options:

Option 1 is to build a large assembly plant. If the economy is strong (probability 40%), you would earn a profit of \$1.25 million; otherwise, you would lose \$500,000.

Option 2 is to build a small assembly plant. If the economy is strong, you would earn a profit of \$1 million; otherwise you would lose \$400,000.

Option 3 is to build nothing. In this case, your profit is \$0, no matter the state of the economy.

10a. For each option, compute the expected value of the profit. Which option should you choose, if you want the maximum possible expected profit?

10b. If you pay me \$150,000, I will predict for you the future state of the economy. (I'm never wrong.) Is this a good deal for you?

11. A gambler enters a casino with a fortune of \$100, and she intends to play a particular game of chance, in which the probability of winning is 50%. When she wins a game, her fortune is doubled; when she loses, her fortune is halved. What is the expected value of her fortune after two successive independent plays of this game?

12. A random variable X has mean 500 and standard deviation 80.

12a. What is the probability that X lies between 100 and 900?

12b. What is the probability that X is more than 400 units away from the mean?

12c. What is the probability that X lies within one standard deviation of the mean?

13. In the last math exam, the mean was 70 and the standard deviation was 6. Find the smallest interval about the mean which is guaranteed to contain at least 90% of the scores.

14. 14a. Let X be the weight of a randomly selected adult. Is X a finite, infinite discrete, or continuous rv?

14b. Suppose an adult is to be selected at random, and their weight is to be measured to the nearest pound. Is this measured weight a finite, infinite discrete, or continuous rv?

14c. In one class, the lightest student weighs 90 lb, the heaviest student weighs 275 lb. Let X be the measured weight of a randomly selected student, where weight is measured to the nearest pound. What is the range of X ?

ANSWERS: WEEK-IN-REVIEW, WEEK #12, M 141, SPRING 2004

1. 1a. missing probability is $P(X = 15) = .20$
 1b. expected value $E(X) = 13$, median = 15, mode = 20
 1c. minimum = 2, $Q_1 = 10$, median = 15, $Q_3 = 20$, maximum = 20
 1d. range = $20 - 2 = 18$, $IQR = Q_3 - Q_1 = 20 - 10 = 10$, variance = 42.4,
 standard deviation = 6.5115

1e.

x	2	4	10	15	20
$P(X \leq x)$	0.15	0.2	0.45	0.65	1.00

1f. $(.35)^3 = .0429$

2. relative-frequency distribution table for “# of children”:

# of children	0	1	2	3	4	5	6
relative frequency	0.319	0.269	0.185	0.151	0.050	0.008	0.017

- 2a. mean = 1.437, median = 1, mode = 0, standard deviation = 1.388, range = 6,
 $Q_1 = 0, Q_3 = 2$
 2b. The probability distribution table is the same as the relative frequency table above;
 simply replace “# of children” with x , replace “relative frequency” with “ $P(X = x)$ ”. The
 summary measures are the same as in problem 2a above.
 2c. $C(38, 2)C(81, 2)/C(119, 4) = .2868$
 2d. $100 \times E(X) = 100 \times 1.437 = 143.7$ children

3. 3a. $P(X = x) = (C(4, x)C(48, (5 - x)))/C(52, 5), x = 0, 1, 2, 3, 4$

x	0	1	2	3	4
p	0.65884	0.29947	0.03993	0.00174	0.00001

- 3b. Expected value (mean) = $5/13 = .38$, median = 0, mode = 0
 variance = .327, standard deviation = .572
 3c. $\$0.38 = \$5/13$

4. 4a.

x	15	20	25	30
p	0.2637	0.4945	0.2198	0.0220

- 4b. Expected value (mean) = 20 cents; median = 20 cents; mode = 20 cents; range =
 15 cents; variance = 14.29 cents²; standard deviation = 3.78 cents
 4c. $200 (\$0.20) = \40.00

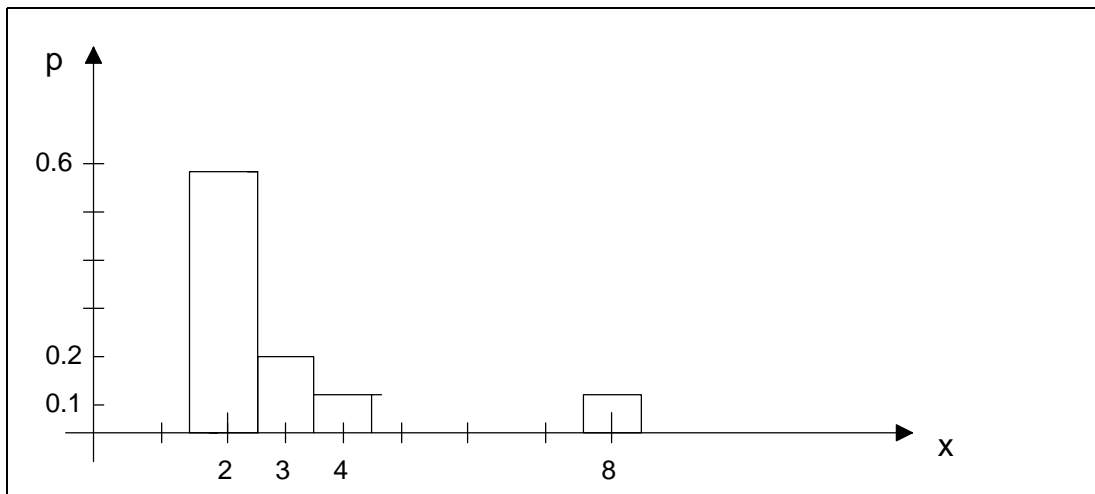
5. 5a. Expected profit = \$3650
 5b. No. Maximum legal premium = \$12,611.11

6. 6a.

x	2	3	4	8
p	0.6	0.2	0.1	0.1

6b. mean = 3, mode = 2, median = 2, standard deviation = 1.79, variance = 3.20, $Q_3 = 3$

6c.



6d. The mean is sensitive to extremes, the median is not. The extreme value of 8 exerts a disproportionate influence on the mean, pulling it above the median.

7. 7a. odds in favor of rain: 3 to 7; odds against rain: 7 to 3

7b. odds in favor of rain: 1 to 4; odds against rain: 4 to 1

8. $5/12 = .417$

9. 9a. Let $X =$ Bob's net winnings, and $a =$ probability that Cal U will win.

x	3	-5
p	a	1 - a

Then $E(X) = 0 = 3(a) + (-5)(1 - a) = 8a - 5$. Solving for a , we get that $a = 5/8$.

(Of course, we could get also this directly: since the odds against Cal are 3 to 5, the probability that Cal wins must be $5/(3+5) = 5/8$.)

9(cont)

9b. Let X = Bob's net winnings, let a = Carl's probability that Cal will win

x	3	-5
p	a	1 - a

Then $E(X) = -3 = 3(a) + (-5)(1-a) = 8a - 5$. Solving for a , we get that $a = 1/4$.

Since Carl's probability that Cal will win is $1/4$, his odds against Cal are 3 to 1, so Al should pay \$3 to Bob for every dollar that Bob bets. Thus, Al should pay Bob $\$3(5) = \15 if Cal wins

Alternatively, we could set up a table which reflects Carl's judgments:

Let X = Bob's winnings, and let b = \$ which Al pays Bob, if Cal wins

x	b	-5
p	1/4	3/4

Then $E(X) = 0 = b(1/4) + (-5)(3/4) = (b-15)/4$. Solving for b , we get that $b = \$15$.

10. 10a. Option 1: Expected profit = $(1,250,000)(.4) + (-500,000)(.6) = \$200,000$

Option 2: Expected profit = $(1,000,000)(.4) + (-400,000)(.6) = \$160,000$

Option 3: Expected profit = $0(.4) + 0(.6) = \$0$.

10b. Choose Option 1. It has the greatest expected profit.

10c. If I predict a strong economy (probability .4), you will build the large plant and earn a profit of $\$1,250,000 - \$150,000 = \$1,100,000$. (The \$150,000 deduction is my fee.)

If I predict a weak economy (probability .6), you will do nothing and lose \$150,000. (You earn no profit from production, but still have to pay my fee.)

The expected profit is $(\$1,100,000)(.4) + (-\$150,000)(.6) = \$350,000$.

You definitely should accept my deal: with "perfect information", your expected profit is \$350,000; without my information, your greatest possible expected profit was only \$200,000.

11. X = fortune after two independent plays

x	400	100	25
p	0.25	0.50	0.25

Expected fortune after two independent plays = \$156.25

12. 12a. at least .96 (Chebyshev, with $k = 5$)

12b. at most .04 (complement of question a)

12c. at least 0 (duh! How enlightening is this answer? Chebyshev, with $k = 1$. Note that Chebyshev's inequality is uninformative unless $k > 1$.)

13. [51,89]. (Use Chebyshev's inequality.)

14. 14a. continuous

14b. finite ("Infinite discrete" would also be a defensible answer, if you didn't care to specify a maximum value in the range of X .)

14c. range of $X = 275.5 - 89.5 = 186$ lb