

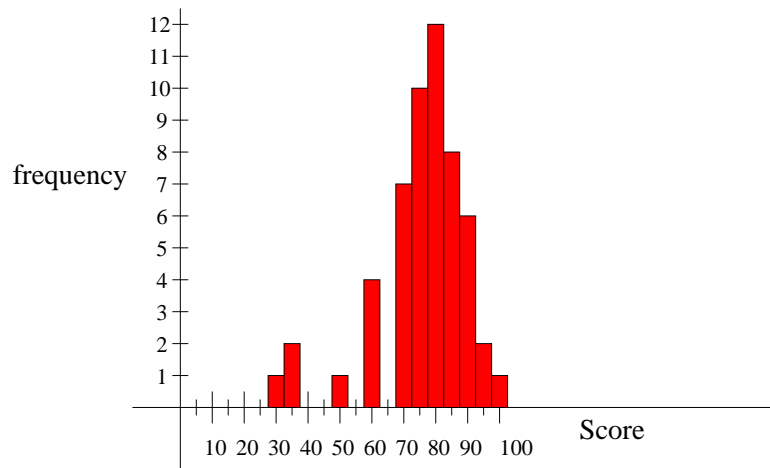
Exam # 3A-Solutions
Math 141.502
Fall 2000

1. [20 pts] An exam was given in a math class, and the following scores were recorded

frequency	1	2	1	4	7	10	12	8	6	2	1
score	30	35	50	60	70	75	80	85	90	95	100

From this data

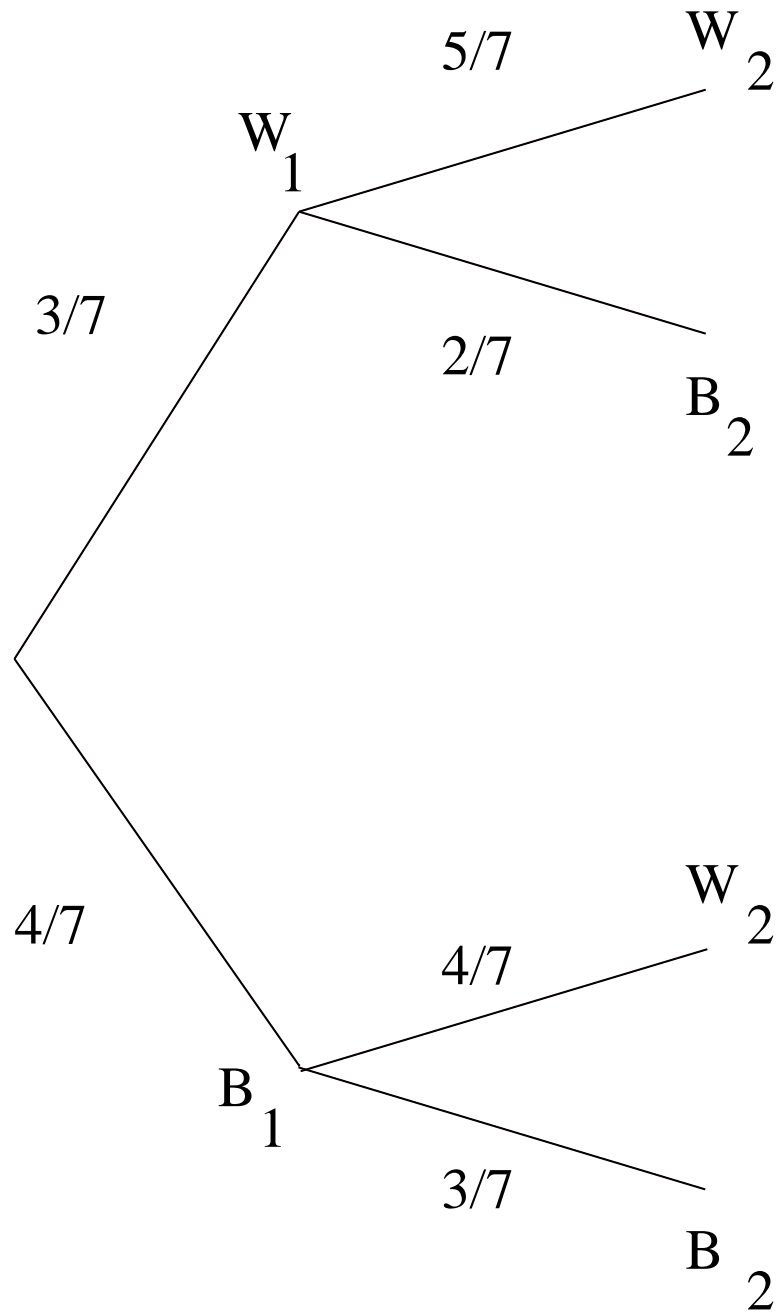
- (a) What is the mean and standard deviation? Using 1-Var Stats on the TI83, $\mu = 75.926$, $\sigma = 14.112$.
- (b) What is the median and mode? Using 1-Var Stats on the TI83, median=80, mode=80.
- (c) What are the first and third quartile scores? $Q_1 = 70$, $Q_3 = 80$
- (d) Plot the histogram for this distribution (use a width of 5 for the vertical bars)



2. [15 pts] Urn A contains 3 white and 4 black balls. Urn B contains 4 white and 2 black balls. A ball is drawn from urn A and then transferred to urn B. A ball is then drawn from urn B. What is the probability that the transferred ball was black, given the second ball drawn was black?

- (a) Draw a tree diagram.

(b) Fill in all the probabilities.



(c) Use Bayes' Theorem to find the answer.

$$P(B_1|B_2) = \frac{(4/7)(3/7)}{(4/7)(3/7) + (3/7)(2/7)} = \frac{12}{18} = 2/3$$

3. [15 pts] 200 TAMU students are chosen for a survey on their dining habits.

It is found that:

90 students eat breakfast at SBISA

120 students eat lunch at SBISA

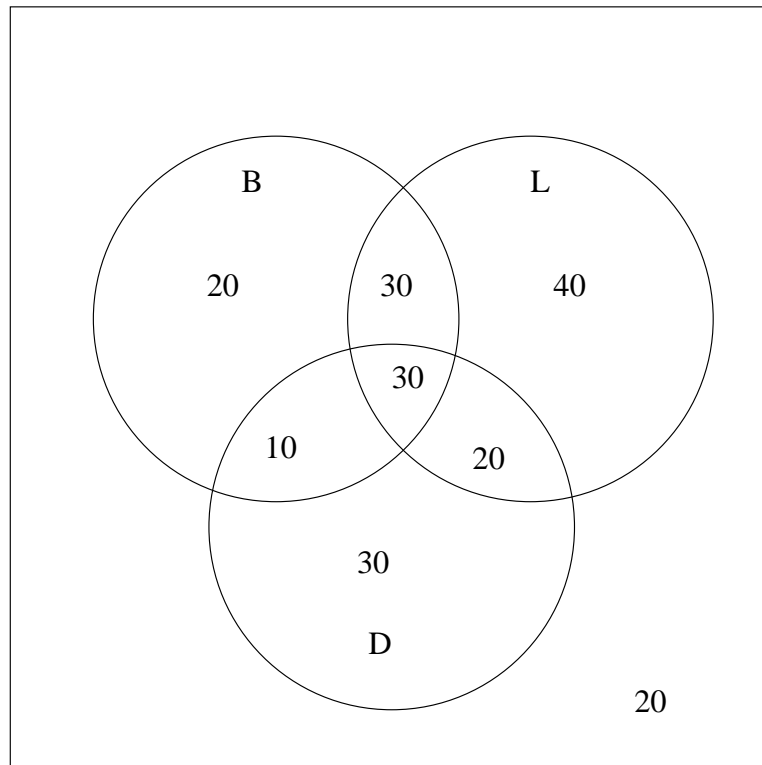
90 students eat dinner at SBISA

60 students eat breakfast and lunch at SBISA

40 students eat breakfast and dinner at SBISA

50 students eat lunch and dinner at SBISA

30 students eat breakfast, lunch and dinner at SBISA



If a student is picked at random from this group, what is the probability that they **do not eat any meals at SBISA**?

Solution There are 20 people who do not eat any meals at SBISA, therefore the probability is $20/200=0.10$, or 10%

4. [10 pts] A fair die is thrown 5 times. What is the probability that it turns up even 2 times and odd 3 times?

Solution There are $C(5,2)=10$ ways to combine 2 evens and 3 odds. Therefore the probability is

$$C(5, 2)\left(\frac{1}{2}\right)^5 = .3125 = 31.25\%$$

5. [10 pts] Given a standard 52 card deck. Let A be the event that a red card is drawn. Let B be the event that a face card is drawn. Are these events **independent**? You must justify your answer!

Solution There are 26 red cards, so $P(A) = 26/52$.

There are 12 face cards, so $P(B) = 12/52$.

There are 6 red face cards, so $P(A \cap B) = 6/52$.

It is easy to show that $P(A \cap B) = 6/52 = 25/52 * 12/52$, so the events are **independent**.

6. [20 pts] As part of a quality control measure, a tire company places series of tires (warrantied to last 40,000 miles) are placed on a testing machine to estimate their lifespan. The amount of mileage before tire failure occurs is measured. The following data was collected:

mileage (in 1000's of miles)	27	30	33	36	40	42	45	50
frequency	1	2	3	5	8	5	5	1

- (a) Can Chebycheff's inequality be used to estimate the percentage of tires lasting between 35,000 and 45,000 miles? If so find the percentage. If not, explain why Chebycheff's inequality cannot be applied.
- (b) Use the actual data to estimate the percentage of tires lasting between 35,000 and 45,000 miles.

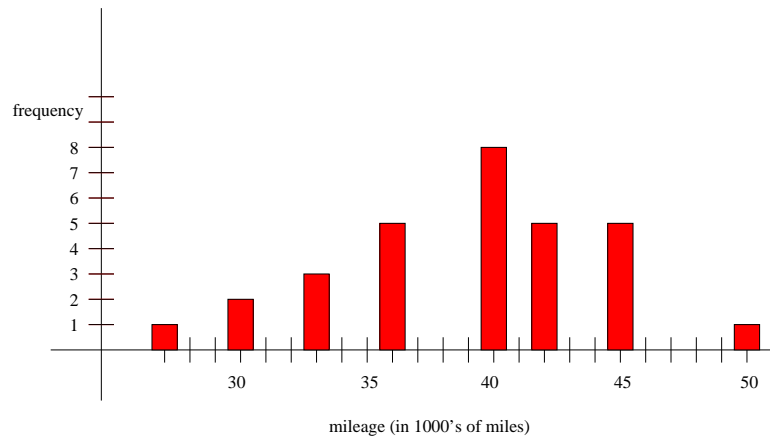
Solution You can answer this in two ways. If you calculate the k value for the left endpoint, you get $k=.774$, which is clearly less than one, so you can not apply Chebycheff directly.

However, If you estimate the k for the right interval, $k=1.15$, you can use it to calculate

$$P(\mu-1.15\sigma < X < \mu+1.15\sigma) = P(33.06 < X < 45.00) \geq 1-1/1.15^2 = 0.240 = 24.0\%$$

so you could apply Chebycheff.

Both answers are acceptable, as long as you provided a correct explanation.



The actual data gives a relative frequency of

$$\frac{(5 + 8 + 5 + 5)}{30} = .766 = 76.6\%$$

7. [10 pts] Explain the difference between the terms *mutually exclusive*, *independent*, *complementary*. You may use an example or illustrations (such as Venn diagrams) to help explain.

Solution *mutually exclusive* events are disjoint, that is their intersection is the empty set.
 $A \cap B = \phi$.

independent events satisfy $P(A \cap B) = P(A) * P(B)$.

complementary events satisfy $A = B^c$.