Exam # 3A

Math 142.518
Spring 2001
(Sections 4.1-4.3, 5.1-5.6)

1. [10 pts] Section 4.1 # 59

   The cost function for direct materials in a furniture factory was approximated by $y = C(x) = 0.667x - 0.01563x^2 + 0.000151x^3$ for $0 \leq x \leq 50$ where x is output in thousands of dollars. Find the approximate value of x for which marginal cost is at a minimum.

   Solution: The marginal cost is the **derivative** of the Cost function, C(x). $C'(x) = 0.667 - 0.03126x + 0.000453x^2$. This is minimized when

   $$C''(x) = -0.03126 + 0.000906x = 0$$

   Therefore, $x = 34.503$ Remember, x is in units of 1000's, so the value is really 34,503 units.

2. [10 pts] Section 5.3 # 9

   If the velocity (as a function of time) is given by $v(t) = \sqrt{(t)}$ find the total distance travelled on $0 \leq t \leq 5$. Estimate the distance by using the midpoint rule with $N = 5$.

   Solution: The exact distance is given by the integral

   $$\int_0^5 \sqrt{t}dt = \frac{2}{3}t^{3/2}|_{t=0}^{t=5} = 7.45356$$

   The approximate distance (via the midpoint formula) is

   $$(\sqrt{0.5} + \sqrt{1.5} + \sqrt{2.5} + \sqrt{3.5} + \sqrt{4.5})(1.0) = 7.50514$$

3. [10 pts] Section 4.1 # 19

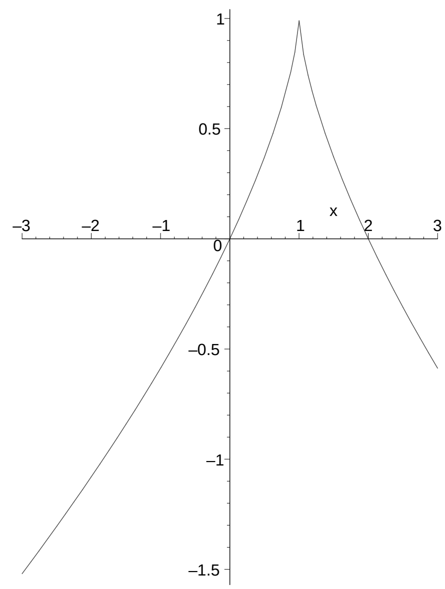
   Find all critical values, the largest open intervals on which f is increasing, the largest open intervals on which f is decreasing, and all relative maxima and minima. Sketch a rough graph of f.

   $$f = 1 - (1 - x)^{2/3}$$

Solution: The critical points are places where $f' = 0$ or f' is **undefined**. Since

$$f'(x) = \frac{2}{3}(1-x)^{-1/2}$$

the only critical point is $x = 1$. $f' > 0$ if $x < 1$ and $f' < 0$ if $x > 1$.



4. [10 pts] Section 4.2 # 59

Find all the critical points and points of inflection for

$$f(x) = x^4 + x^3 + x^2 + x + 1$$

Sketch the curve.

Solution: The critical points and inflection points are found by examining

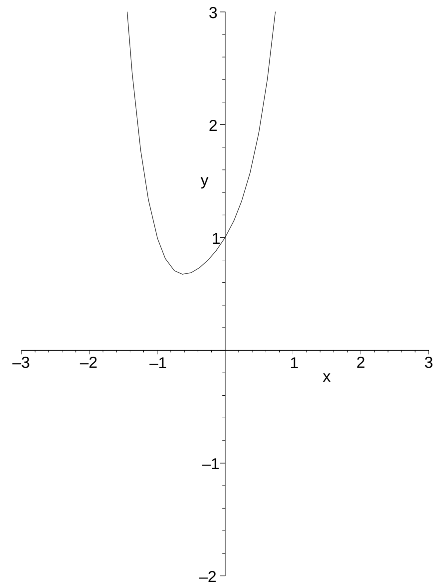
$$f'(x) = 4x^3 + 3x^2 + 2x + 1 = 0$$

and

$$f'(x) = 12x^2 + 6x + 2 = 0$$

To find the zeros of f' we use the [2nd][calc][zero] command on the TI83. This gives us $x = -.6058296$ as the critical point. The inflection points are given by the **quadratic formula** which in this case tells us there are no real roots (square root of a negative number) and therefore no inflection points.

The graph is below



5. [10 pts] Section 5.1 # 47

The height of a ball, initially at h_0 feet above the ground, thrown with velocity v_0 feet per second, is given by

$$h(t) = h_0 + v_0 t - 16t^2$$

Before it hits the ground, find the maximum velocity, and maximum height, a ball achieves when it is thrown up into the air at a rate of 50 ft/sec from a a height of 6 feet.

Solution: Substituting the values from the problem

$$h(t) = 6 + 50t - 16t^2$$

The maximum height is achieved when $h'(t) = 50 - 32t = 0$, or $t = 50/32$. At this time, $h_{max} = h(50/32) = 45.0625$.

Since the velocity is $v(t) = h'(t) = 50 - 32t$, we have $v'(t) = -32$ which never vanishes. This means the maximum must occur at the end points. When $t = 0$, $v = v(0) = 50$. When it hits the ground, $h = 0$ or $6 + 50T - 16T^2 = 0$ or $T = 3.24$ seconds. For this time, $v = v(T) = -53.68$. The maximum velocity is 53.68 ft/sec.

6. [10 pts] Section 5.2 # 33

Find the indefinite integral

$$\int \frac{1}{x(\ln x)^2} dx$$

Solution: This is solved by a **change of variable** $u = \ln x$. In this case, $du = 1/x dx$ so the integral reduces to

$$\int \frac{du}{u^2} = -\frac{1}{u} + C = -\frac{1}{\ln|x|} + C$$

7. [10 pts] Section 5.5 # 22

Evaluate the definite integral

$$\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$$

Solution: This is solved by a **change of variable** $u = \sqrt{x}$. In this case, $du = 1/(2\sqrt{x})dx$, and

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = \int 2e^u du = 2e^u + C = 2e^{\sqrt{x}} + C$$

so

$$\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = 2e^{\sqrt{x}}|_{x=1}^{x=4} = 2e^2 - 2e^1 = 9.341548541$$

This can also be done via the TI83 programs prgmRiemann(), and FnInt().

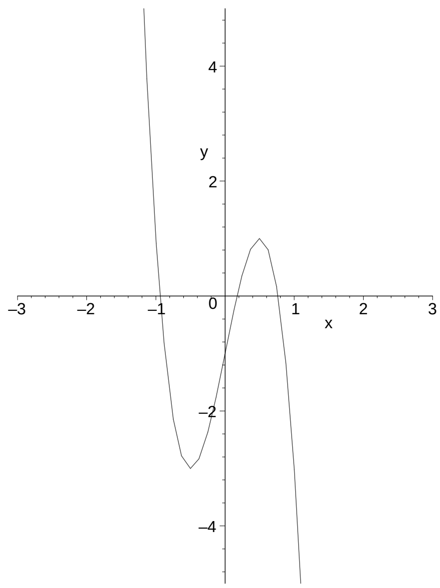
8. [10 pts] Section 4.3 # 18

Locate the value(s) at which each function attains an absolute maximum and an absolute minimum, if they exist, of the given function on the given interval.

$$f(x) = -8x^3 + 6x - 1 \text{ on } [-1, 1]$$

Solution: The critical points are found by examining $f'(x)$. $f'(x) = -24x^2 + 6 = 0$ when $x^2 = 6/24 = 1/4$ so $x = \pm 1/2$. We also look at the endpoints, and conclude $f(-1) = 1$, $f(-1/2) = -3$, $f(1/2) = 1$ and $f(1) = -3$. Therefore there are absolute maxima at $x = -1, 1/2$ and absolute minima at $x = -1/2, 1$.

The graph is below:



9. [10 pts] Section 5.1 # 37

   Find the anti-derivative

$$\int \frac{x+1}{x} dx$$

   Solution: As a general rule, **simplify first** then **change of variables** then integrate. We have

$$\int \frac{x+1}{x} dx = \int (1 + \frac{1}{x}) dx = x + \ln x + C$$

10. [10 pts] Section 5.6 # 6

   Find the area enclosed by the curves $y = \sqrt{x}$ and $y = x$ on [0,1]

   Solution: This is given by the difference of the two integrals

$$\int_0^1 \sqrt{x} dx - \int_0^1 x dx = \frac{2}{3} x^{3/2} |_{x=0}^{x=1} - \frac{1}{2} x^2 |_{x=0}^{x=1} = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$$