What are the odds?

1. Royal Flush. This consists of a {10,J,Q,K,A} of the same suit. The number of ways that this can come about is (number of ways of choosing 1 suit from 4 suits) $\times$ (number of ways to choose {10,J,Q,K,A}) = C(4,1) $\times$ C(5,5) = 4 $\times$ 1 = 4.

2. Straight Flush. This consists of 5 cards in a row of the same suit. The number of ways that this can come about is (number of ways of choosing 1 suit from 4 suits) $\times$ (number of ways of choosing 5 cards in a row) = C(4,1) $\times$ (10) = 40.

3. Four of a Kind. The number of ways that this can come about is (number of ways of choosing 1 rank from 13 ranks) $\times$ (number of ways of choosing 4 cards from 4) $\times$ (number of ways of choosing the fifth card) = 13 $\times$ C(4,4) $\times$ C(48,1) = 13 $\times$ 1 $\times$ 48 = 624.

4. Full House. This consists of three of a kind and two of a kind. The number of ways that this can come about is (number of ways of choosing 1 rank from 13) $\times$ (number of ways of choosing 3 cards from 4) $\times$ (number of ways of choosing 1 rank from 12) $\times$ (number of ways of choosing 2 cards from 4) = C(13,1) $\times$ C(4,3) $\times$ C(12,1) $\times$ C(4,2) = 13 $\times$ 1 $\times$ 12 $\times$ 6 = 3,744.

5. Flush. This consists of 5 cards of the same suit, but not in sequence. The number of ways that this can come about is (number of ways of choosing 1 suit from 4) $\times$ (number of ways of choosing 5 cards from 13 in a suit) - (number of straight flushes)= C(4,1) $\times$ C(13,5) - 40 = 4 $\times$ C(13,5) - 40 = 5,108.

6. Straight. This consists of 5 cards in a row, but not of the same suit. The number of ways that this can come about is (number of ways of choosing 5 values in a row) $\times$ (number of ways of choosing first card from 4) $\times$ (number of ways of choosing second card from 4) $\times$ (number of ways of choosing third card from 4) $\times$ (number of ways of choosing fourth card from 4) $\times$ (number of ways of choosing fifth card from 4) - (number of straight flushes)= 10 $\times$ C(4,1)^5 - 40 = $4^5 \times 10 - 40$ = 10,200.

7. Three of a Kind (but not full house, or four of a kind). The number of ways that this can come about is (number of ways of choosing 1 rank from 13) $\times$ (number of ways of choosing 3 cards from 4) $\times$ (number of ways of chosing 1 card from rermaining 48) $\times$ (number of ways of chosing 1 card from rermaining 47) - (number of four of a kin) - (number of full house) = C(13,1) $\times$ C(4,3) $\times$ C(49,1) $\times$ C(48,1) - 624 - 3744 = 13 $\times$ 4 $\times$ 49 $\times$ 48 - 624 - 3744 = 117,936.

8. Two Pair (but not four of a kind). The number of ways that this can come about is (number of ways of choosing two different ranks from 13) $\times$

(number of ways of choosing two cards from four) × (number of ways of choosing two cards from four) × (number of ways of choosing remaining card) = C(13,2) × C(4,2) × C(4,2) × C(44,1) = 78 × 6 × 6 × 44 = 123,552.

9. One Pair (but not two pair, or three of a kind or full house). The number of ways that this can come about is (number of ways of choosing 1 rank from 13) × (number of ways of choosing two cards from four) × (number of ways of choosing another rank) × (number of ways of choosing one card from four) × (number of ways of choosing another rank) × (number of ways of choosing one card from four) × (number of ways of choosing another rank) × (number of ways of choosing one card from four) = C(13,1) × C(4,2) × C(12,1) × C(4,1) × C(11,1) × C(4,1) × C(10,1) × C(4,1) / 3! = 13 × 6 × 12 × 4 × 11 × 4 × 10 × 4 = 1,098,240. ×

The total number of 5 card combinations are C(52,5) = 2,598,960.
The probabilities are therefore:

1. Royal Flush. 4 in 2,598,960.
2. Straight Flush. 40 in 2,598,960.
3. Four of a Kind. 624 in 2,598,960.
4. Full House. 3,744 in 2,598,960.
5. Flush. 5,108 in 2,598,960.
6. Straight. 10,200 in 2,598,960.
7. Three of a Kind. 117,936 in 2,598,960.
8. Two Pair 123,552 in 2,598,960.
9. One Pair 1,098,240 in 2,598,960.