

Math 304  
Quiz #8B - Solution  
Sections 5.3,5.4

1. [8 pts] Find the best quadratic least squares fit to the data

x	0	1	2	3	4
y	0	-1	2	5	10

**Solution:** The matrix  $A$  is given by

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \end{bmatrix}$$

Therefore, least squares equations are  $A^T A x = A^T b$  or

$$x = \begin{bmatrix} 5 & 10 & 30 \\ 10 & 30 & 100 \\ 30 & 100 & 354 \end{bmatrix}^{-1} \begin{bmatrix} 16 \\ 58 \\ 212 \end{bmatrix}$$

which has the solution  $-0.2857 - 0.8286x + 0.8571x^2$

2. [7 pts] Find the angle between 1 and  $x$  using the weighted inner product

$$\langle u, v \rangle = \int_0^1 x u(x) v(x) dx$$

**Solution:** The inner product of 1 and  $x$  is given by

$$\langle 1, x \rangle = \int_0^1 x^2 dx = \frac{1}{3}$$

The lengths of each vector are given by

$$\|1\|^2 = \int_0^1 x dx = \frac{1}{2} \Rightarrow \|1\| = \frac{1}{\sqrt{2}}$$

$$\|x\|^2 = \int_0^1 x^3 dx = \frac{1}{4} \Rightarrow \|x\| = \frac{1}{2}$$

The angle (in radians) is given by

$$\cos(\theta) = \frac{1/3}{(1/\sqrt{2})(1/2)} = \frac{2\sqrt{2}}{3} = 0.3398$$