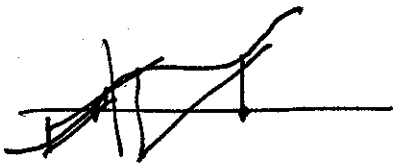
Chapter 1.

Linear Equations — Solve "exactly"

Systems — more than one unknown.



$$f(x) = f(x_0) + f'(x_0)(x - x_0)$$
$$= mx + b \leftarrow$$

when does lin eq^n have sol?

how to compute it?

$Ax = b$ has a sol if A^{-1} exists. (A square)

if $A_{m \times n}$, $m \neq n \Rightarrow$ least square!

$A^T A x = A^T b$ — rref ...

$x = (A^T A)^{-1} A^T b$ least sq. sol.

* RREF (TI-83...) — lead variable → cols with "1"
free variable → everything else!

* create matrices, vectors, $\{+, -, \times, "-1"\}$
(real coefficients)

row reduction ↪ gaussian elim ↪ gauss-jordan method.

Ch. 2 Determinants

minors
cofactors → know how to set these up

Cramer's rule.

Chapter 3 - Vector Spaces. (VS)

extensions of $\mathbb{R}^N$ ($\mathbb{R}^2, \mathbb{R}^3, \ldots$)

need A1–A8 (p. 119) to solve equations

Subspaces – subsets of V.S. with closure ($+/-$, $\times$)

* Show closure!

Important Subspaces – A, A^T
- Row space
- Column space (Range)
- Null space

* Show set is lin independent!

if $c_1\vec{x}_1 + c_2\vec{x}_2 + \cdots c_N\vec{x}_N = \vec{0} \Rightarrow c_1=0, c_2=0, \ldots, c_N=0$ $\Rightarrow$ linearly indep.

$\Rightarrow$ basis.

* Wronskian
* rank, dimension
* change bases. $[E]x_E = [F]x_F \Rightarrow x_E = E^{-1}F\,x_F$ $F \to E$

$x_F = F^{-1}E\,x_E$ $E \to F$

Chapter 4 Linear Transformations

* Verify when something is linear. $L[x+y] = L[x] + L[y]$, $L[cx] = cL[x]$
* representation by matrix. $L[x] = y$

$[L]x_E = y_F$

($E = F$) * Similarity

Chapter 5 Orthogonality (angles / distances)

* inner product $= \langle u, v \rangle$, $\Rightarrow \|u\| = \langle u, u \rangle^{1/2}$

$$\Rightarrow \cos\theta = \frac{\langle u, v \rangle}{\|u\| \, \|v\|}$$

* scalar projections
* vector projections
* orthogonal subspaces (null, ~~sp~~ range)
* Least squares!
* orthogonal, orthonormal sets!

(5.6, 5.7 not on final)

Chapter 6 E-value, E-vectors (e-functions)

↓ frequencies ↓ modes of vibration

$$Ax = \lambda x$$

$$\det(A - \lambda I) = 0 \Rightarrow \gamma_N(\lambda) \Rightarrow N \text{ roots}$$

$$\lambda \longleftrightarrow x_\lambda$$

e-val e-vectors

diagonalization (similarity)