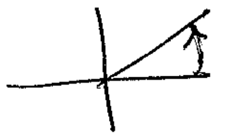
209 #6

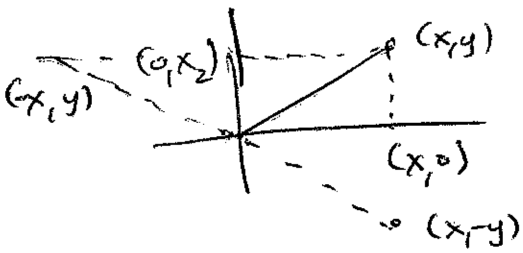
$L: \mathbb{R}^2 \to \mathbb{R}^2$ $\quad \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ rotates vector in $\mathbb{R}^2$ by "θ" in circlockwise direction



30°

$\begin{bmatrix} \cancel{\sin\theta} & \cos\theta \\ \cancel{\cos\theta} & \end{bmatrix}$

$\begin{bmatrix} \cos\theta & +\sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ $\quad \theta \quad \circlearrowright$



$L\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ 0 \end{bmatrix}$ projection onto x_1-axis

$L\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ x_2 \end{bmatrix}$ " " x_2-axis

$L\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ -x_2 \end{bmatrix}$ reflection thru x_1-axis

$L\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -x_1 \\ x_2 \end{bmatrix}$ " " x_2-axis

$L\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -x_1 \\ -x_2 \end{bmatrix}$ reflection wrt. origin

$$L\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$= \begin{bmatrix} x_1\cos\theta - x_2\sin\theta \\ x_1\sin\theta + x_2\cos\theta \end{bmatrix} \quad \theta \circlearrowleft$$

208 #3

$L: \mathbb{R}^3 \to \mathbb{R}^3$

$$L\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_2 - x_1 \\ x_3 - x_2 \\ x_3 - x_1 \end{bmatrix}$$

$$S = \text{span}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \left\{ \begin{bmatrix} \alpha \\ 0 \\ \alpha \end{bmatrix} = \alpha \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$$

ker(L) $\Rightarrow$

$$L\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} x_2 - x_1 &= 0 \\ x_3 - x_2 &= 0 \\ x_3 - x_1 &= 0 \end{aligned} \Rightarrow \begin{aligned} x_2 &= x_1 \\ x_3 &= x_2 \\ x_3 &= x_1 \end{aligned}$$

$\cancel{x_1 = \alpha, \Rightarrow x_2 = \alpha, x_3 = \alpha}$

$$\ker L = \text{span}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$L\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}$$

$$L\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \to \begin{bmatrix} -1 & +1 & 0 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$L\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\det = -1 \begin{vmatrix} -1 & 1 \\ -1 & 0 \end{vmatrix} + 1 \begin{vmatrix} -1 & 1 \\ 0 & -1 \end{vmatrix} = (-1)(+1) + 1(1) = -1 + 1 = 0$$

null (A)

$$\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

b) $L[S] = L\begin{bmatrix} \alpha \\ 0 \\ \alpha \end{bmatrix} = \begin{bmatrix} -\alpha \\ \alpha \\ 0 \end{bmatrix} = \alpha \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = \text{span}\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$

$\{u_1, u_2, \ldots, u_N\}$ = basis for S

$$L[S] = L[c_1 u_1 + c_2 u_2 + \cdots c_n u_n] = c_1 \underset{\substack{\parallel \\ v_1}}{L[u_1]} + \cdots c_n \underset{v_n}{L[u_n]}$$

2 08 #2

$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$

$L[v_1] = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$ $L[v_2] = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$ $L[v_3] = ?$

$\vec{v}_3 = c_1\vec{v}_1 + c_2\vec{v}_2$, $c_1 = ?$ $c_2 = ?$

$$\begin{bmatrix} 1 \\ 7 \end{bmatrix} = c_1\begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} c_1 - c_2 \\ c_1 + 2c_2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}^{-1}\begin{bmatrix} 1 \\ 7 \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$, or $\left[\begin{array}{cc|c} 1 & -1 & 1 \\ 1 & 2 & 7 \end{array}\right] \leftarrow$ rref

$\rightarrow c_1 = 3.0$
$c_2 = 2.0$

$$3\begin{bmatrix} 1 \\ 1 \end{bmatrix} + 2\begin{bmatrix} -1 \\ 2 \end{bmatrix} \stackrel{?}{=} \begin{bmatrix} 1 \\ 7 \end{bmatrix}$$

$$L\begin{bmatrix} 1 \\ 7 \end{bmatrix} = 3L\begin{bmatrix} 1 \\ 1 \end{bmatrix} + 2L\begin{bmatrix} -1 \\ 2 \end{bmatrix}$$
$$= 3\begin{bmatrix} 2 \\ 5 \end{bmatrix} + 2\begin{bmatrix} -3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 17 \end{bmatrix}$$

4

p. 205 #5

$L: \mathbb{P}_3 \to \mathbb{P}_3$

$L[p(x)] = xp'(x) + p''(x)$

A) $\{1, x, x^2\} = E$

$L[1] = 0$
$L[x] = x$
$L[x^2] = 2x^2 + 2$

$$[L]_E = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} = A$$

B) $\{1, x, 1+x^2\} = F$

$L[1] = 0$
$L[x] = x$
$L[1+x^2] = 2x^2 + 2 = 2(x^2+1)$

$$\to \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \quad B$$

$2x^2 + 2 = c_1 \cdot 1 + c_2 \cdot x + c_3(1+x^2)$

$c_1 = 0, \; c_2 = 0 \quad c_3 = 2$

c) $B = S^{-1}AS \quad S?$

$L[x] = y$

$A[x]_E = [y]_E$

$B[x]_F = [y]_F$

$E \cdot [x]_E = F[x]_F$

$A[F][x]_F = F[y]_F$

$\underbrace{F^{-1}AF}_{B} [x]_F = [y_F]$

$$S = F = \begin{array}{c} \begin{array}{ccc} 1 & x & 1+x^2 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$[L]_E [x]_E = [y]_E$$

$$[L]_F [x]_F = [y]_F$$

$$E[x]_E = F[x]_F \qquad [x]_E = E^{-1}F[x_F] \qquad F \to E$$

$$E[y]_E = F[y]_F \Rightarrow [y]_F = F^{-1}E[y]_E \qquad E \to F$$

$F^{-1}E =$ C.O.B from E to F.

$$[L]_E E^{-1}F[x]_F = E^{-1}F[y]_F$$

$$\underbrace{F^{-1}E[L]_E E^{-1}F}_{[L]_F} [x]_F = [y]_F$$

$$[L]_F = F^{-1}E[L_E]E^{-1}F$$

$$\boxed{F[L]_F F^{-1} = E L_E E^{-1}}$$

270 #1c)

$$[1,-1]^T\begin{bmatrix}1\\1\end{bmatrix} = 1-1 = 0 \checkmark \text{ orthog}$$

$$\|[1,-1]\| = \sqrt{1^2+1^2} = \sqrt{2}$$

$$\left\{ \frac{1}{\sqrt{2}}[1,-1]^T,\ \frac{1}{\sqrt{2}}[1,1]^T \right\}$$

243 #3a)

$$A = \begin{bmatrix}1 & 2\\ 2 & 4\\ -1 & -2\end{bmatrix}_{3\times 2} \quad b = \begin{bmatrix}3\\2\\1\end{bmatrix}_{3\times 1} \quad A^TA\,x = A^Tb$$

det $A = 0$!
rank $A = 1 < 2$
$\Rightarrow A^TA$ singular!

$$\begin{bmatrix}6 & 12\\ 12 & 24\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}_{2\times 1} = \begin{bmatrix}6\\12\end{bmatrix}$$

$$\left[\begin{array}{cc|c}1 & 2 & 1\\ 0 & 0 & 0\end{array}\right]$$

$x_1 + 2x_2 = 1$

$x_2 = \alpha$

$x_1 = 1 - 2\alpha$

$x_2 = \alpha$

$$\boxed{\begin{bmatrix}1-2\alpha\\ \alpha\end{bmatrix}}$$

2#1 #7

$\{u_1, u_2, u_3\}$ = o-n basis.

$$\vec{x} = c_1\vec{u}_1 + c_2\vec{u}_2 + c_3\vec{u}_3$$

a) $\|x\| = 5 \rightarrow$ $\|x\|^2 = \boxed{c_1^2 + c_2^2 + c_3^2 = 5^2}$

b) $\langle u_1, x\rangle = 4$

c) $x \perp u_2 \Rightarrow \langle x, u_2\rangle = 0$

$c_1 = ?, c_2?, c_3?$

$4 = \langle x, u_1\rangle = c_1\langle u_1, u_1\rangle + c_2\langle u_2, u_1\rangle + c_3\langle u_3, u_1\rangle = c_1$

$c_1 = 4$

$0 = \langle x, u_2\rangle = c_1\langle u_1, u_2\rangle + c_2\langle u_2, u_2\rangle + c_3\langle u_3, u_2\rangle = \boxed{c_2 = 0}$

$c_1 = 4$
$c_2 = 0$
$c_1^2 + c_2^2 + c_3^2 = 5^2$
$16 + 0 + c_3^2 = 25$
$c_3^2 = 9$, $\boxed{c_3 = \pm 3}$

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 3 \end{bmatrix}$$

233 #3

$$S = \text{span}\left\{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}\right\}$$

$$A = \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{bmatrix}$$

$$\text{null}(A) = [A]z = 0 \Rightarrow \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow [x_1, x_2, x_3] \perp [z_1, z_2, z_3]$$
$$[y_1, y_2, y_3] \perp [z_1, z_2, z_3]$$

$$z \in \text{Null}(A) \Rightarrow z \perp \vec{x}, \vec{y} \Rightarrow z \in S^\perp$$

$$z \in S^\perp \Rightarrow z \in \text{Null}(A)$$

$$\Rightarrow \text{Null}(A) = S^\perp$$

b) $S = \text{span}\left(\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}\right)$ $\quad S^\perp$

$\dim(S) = 2$
$\dim(S^\perp) = 1$

$[a, b, c] \perp [1, 2, 1]$
$[a, b, c] \perp [1, -1, 2]$ $\Rightarrow$ $a + 2b + c = 0$, $a - b + 2c = 0$

$c = 1$

$$\begin{aligned} a + 2b &= -1 \\ -\quad a - b &= -2 \\ \hline 0 \quad 3b &= 1 \end{aligned}$$

$b = 1/3$, $a = -5/3$

$[-5/3, 1/3, 1]$

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

P-233 1d) $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 2 & 2 \end{bmatrix}$

$\text{rref}(A) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ rank $= \cancel{2}$ 3

① $R(A) = \text{span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 2 \end{bmatrix} \right\}$ NOT $\cancel{\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}}$

dim $= 3$

② $N(A)$ $x_4 = \alpha$, $x_1 = 0$, $x_2 = 0$, $x_3 + x_4 = 0$, $0 = 0$ $x_3 = -x_4 = -\alpha$ $\begin{bmatrix} 0 \\ 0 \\ -\alpha \\ \alpha \end{bmatrix} = \alpha \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix}$

$= \text{span} \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix}$

dim $= 1$

③ $R(A^T) = $ row space $A =$

span $\{[1\ 0\ 0\ 0], [0, 1, 0, 0], [0, 0, 1, 1]\}$

④ $N(A^T)$ $\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \rightarrow$ $a + d = 0$, $b + d = 0$, $b + c + 2d = 0$ $d = \alpha$, $a = -\alpha$, $b = -\alpha$, $c = -2\alpha + \alpha = -\alpha$

$\begin{bmatrix} -\alpha \\ -\alpha \\ -\alpha \\ \alpha \end{bmatrix}$ $\alpha = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$

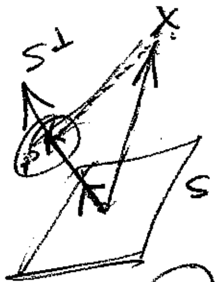
A^T

$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ ✓

p-296 #6

$$S = \text{span}\left\{ \begin{bmatrix}1\\0\\2\end{bmatrix}, \begin{bmatrix}0\\+1\\-2\end{bmatrix} \right\}$$

a) $\dim(S^\perp) = 1$, $\quad \begin{bmatrix}a\\b\\c\end{bmatrix} \perp \begin{bmatrix}1\\0\\2\end{bmatrix}, \begin{bmatrix}0\\1\\-2\end{bmatrix}$



$a + 2c = 0 \quad b - 2c = 0$

$c = 1, \; a = -2, \; b = 2$

$$S^\perp = \text{span}\left\{ \begin{bmatrix}-2\\2\\1\end{bmatrix} \right\}$$

proj X onto $S^\perp$

$\Rightarrow$ onto $\begin{bmatrix}-2\\2\\1\end{bmatrix} = \vec{p}$

$$\frac{\langle x, p \rangle}{\|p\|^2} p$$

$$\left(\frac{\langle x, p \rangle}{\|p\|}\right) \frac{p}{\|p\|}$$

(Projection Matrix given by p. 262 Corollary 5.5.9)

p. 204 #2)

$$[U] = [u_1 | u_2] = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$V = [v_1, v_2] = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$$

$$L[x] = L\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -x_1 \\ x_2 \end{bmatrix}$$

$$B = [L]_U$$

$$L[u_1] = L\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \vec{u}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$L[u_2] = L\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} = u_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\cancel{[L]_U = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}}$$

$$[L]_U = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = B$$

a) $S: U \to V$

$$U[x]_U = V[X]_V$$

$$V^{-1}U[x]_U = [X]_V$$

$$S = V^{-1}U = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = S = \begin{bmatrix} 1 & 1 \\ -1 & -3 \end{bmatrix}$$

$V \to U$

$$U^{-1}V[X]_V = [X]_U$$

$$S^{-1} = \begin{bmatrix} 3/2 & 1/2 \\ -1/2 & -1/2 \end{bmatrix}$$

b) $V[L]_V V^{-1}$

$= U[L]_U U^{-1}$

$$\cancel{[L]_U = U^{-1}V}$$

$$[L]_V = V^{-1}U[L]_U U^{-1}V$$

$$= S[L]_U S^{-1} = SBS^{-1} = \begin{bmatrix} 1/2 & -1/2 \\ -1/2 & -1/2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ -4 & -1 \end{bmatrix}$$

$u_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$u_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$\vec{u} = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} \to \begin{bmatrix} c_1 - c_2 \\ c_1 + c_2 \end{bmatrix}$

$= c_1 \vec{u}_1 + c_2 \vec{u}_2$

$= \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$

$\quad u_1 \quad u_2$

$\vec{u} = \underbrace{\left[\vec{u}_1 \,\middle|\, \vec{u}_2 \right]}_{[U]} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$ $\uparrow$ $[x]_u$

$S = \mathrm{span}\{\vec{u}_1, \vec{u}_2\}$

$S^\perp \quad v_1 \perp u_1$

$v_1 \perp u_2$

$\exists\, v_2 \perp v_1, u_1, u_2$

$\vec{u} = [U][x]_u$

p-233 #4

$S = \mathrm{span}\left\{ \begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 3 \\ -2 \end{bmatrix} \right\}$ $\quad \begin{bmatrix} 2 \\ -3 \\ 1 \\ 0 \end{bmatrix} \perp$

$(a, b, c, d) \perp$

$d = 0$

$a - 2c + d = 0$

$b + 3c - 2d = 0$

$a - 2c = 0 \qquad c = 1,\ a = 2,\ b = -3$

$b + 3c = 0$

$(a, b, c, d) \perp$

$a - 2c + d = 0$

$b + 3c - 2d = 0$

$2a - 3b + c = 0$

p. 252 #7

$C[0,1] \to \int_0^1 u(x)v(x)\,dx$

a) $\langle e^x, e^{-x} \rangle = \int_0^1 e^x e^{-x}\,dx = \int_0^1 1\,dx = 1$

b) $\langle x, \sin \pi x \rangle = \int_0^1 x \sin \pi x\,dx =$ I.B.P. . .

c) $\langle x^2, x^3 \rangle = \int_0^1 x^5\,dx = \frac{1}{6}$